WAVELET-BASED BAYESIAN ESTIMATOR FOR POISSON NOISE REMOVAL FROM IMAGES

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ABSTRACT

Images are, in many cases, degraded even before they are encoded. Emission and transmission tomography images, X-ray films, and photographs taken by satellites are usually contaminated by quantum noise, which is Poisson distributed. Poisson shot noise is a natural generalization of a compound Poisson process when the summands are stochastic processes starting at the points of the underlying Poisson process. Unlike additive Gaussian noise, Poisson noise is signal-dependent and separating signal from noise is a difficult task. A wavelet-based maximum likelihood for a Bayesian estimator that recovers the signal component of the wavelet coefficients in original images by using an alpha-stable signal prior distribution is extended to the Poisson noise removal from a previous investigation. As we discussed in our earlier papers that Bayesian estimator can approximate impulsive noise more accurately than other models and that in the general case the Bayesian processor does not have a closed-form expression. The parameters relative to Bayesian estimators of the model are carefully investigated after an investigation of α-stable simulations for a maximum likelihood estimator. As an example, an improved Bayesian estimator that is a natural extension of other wavelet denoising (soft and hard threshold methods) via a colour image is presented to illustrate our discussion.

1. INTRODUCTION

It is well known that noise degrades the performance of any image compression algorithm. In many cases, degraded even before they are encoded. It is obvious that linear filtering techniques used in many image-processing applications, are attractive due to their mathematical simplicity, and efficiency in the presence of additive Gaussian noise. However, they also blur sharp edges, make some distortions of lines and fine image details, less effectively remove tailed noise, and poorly treat the presence of signal-dependent noise. For example, emission and transmission tomography images are usually contaminated by quantum noise, which is Poisson noise. Unlike additive Gaussian noise, Poisson noise is signal-dependent, and separating signal from noise is difficult. Several groups have discussed that wavelet subband coefficients have highly non-Gaussian statistics [1-6] and the general class of α-stable distributions has also been shown to accurately model heavy-tailed noise [5-7]. The question is, how to deal with α-stable distributions with signal-dependent noise, such as signals contaminated by Poisson noise.

Wavelet transform is a powerful tool for recovering signals from noise and has been of considerably interest [4, 8-11]. In fact, wavelet theory combines many existing concepts into a global framework and hence becomes a powerful tool for several domains of application.

As mentioned by Achim *et al.* [12], there are two major drawbacks for thresholding. One is that choice of the threshold is always done in an *ad hoc* manner; another is that the specific distributions of the signal and noise may not be well matched at different scales.

Donoho gives some minimum thresholds for several threshold schemes, titled "universal thresholds" [10]. These explicitly depend on the standard deviation of noise, where the standard deviation is assumed to be known. In practice, the standard deviation can be readily estimated using the methods discussed in [9], [13]. For some applications the optimal threshold can be computed. An approach different from "universal thresholds" is presented by Nason [14], in which cross-validation is used. Two approaches to cross validation are used, namely ordinary cross validation (OCV) and generalised cross validation (GCV): each is used to minimize the least-squares error between the original (which is the unknown value) function and its estimate based on the noisy observation.

Modelling the statistics of raw images is a challenging task because of the high dimensionality of the signal and the complexity of statistical structures that are prevalent. Numerous papers discuss modelling the statistics of raw images, including Bayesian processing assuming proper modelling of the prior probability density function of the signal, but they dealt with the Gaussian noise, or with the symmetric stochastic distributions [5, 6, 15,16,17,19,20].

In this paper it is carefully discussed that a wavelet-based maximum likelihood for Bayesian estimator that recovers the signal component of the wavelet coefficients in original images from images contaminated by Poisson noise, by using an alpha-stable signal prior distribution.

As an example, a colour image and its image contaminated by Poisson noise will be shown using the discussed method.

2. ALPHA-STABLE DISTRIBUTIONS AND LOG LIKELIHOOD

It is well known that the symmetric alpha-stable distribution ($S\alpha S$) distribution is defined by its characteristic function:

$$\phi(\omega) = \exp(j\delta\omega - \gamma\,|\,\omega\,|^{\alpha}), \tag{1}$$

The parameters α, γ, and δ describe completely a $S\alpha S$ distribution. The characteristic exponent α controls the heaviness of the tails of the stable density. α can take values in (0,2]; while α = 1 and 2 are the Cauchy and Gaussian cases respectively. There is not closed-form expression known for the general $S\alpha S$ probability density function (PDF). Thus, it is useful when using the principle of maximum likelihood estimation. The dispersion parameter γ (γ>0) refers to the spread of the PDF. The location parameter δ is analogous to the mean of the PDF, which, for our following discussion, will be the same assumption as that in [5].

If a variable $\hat{\theta}$ is unbiased it follows that

$$E(\hat{\theta}-\theta)=0 \tag{2}$$

which can be expressed as:

$$\int_{-\infty}^{\infty}\cdots\int(\hat{\theta}-\theta)f_{\vec{x};\theta}(\vec{x};\theta)d\vec{x}=0 \tag{3}$$

where $\vec{x}(\xi)=[x_1(\xi),x_2(\xi),\ldots,x_N(\xi)]^T$ and $f_{\vec{x};\theta}(\vec{x};\theta)$ is the joint density of $\vec{x}(\xi)$, which depends on a fixed but unknown parameter. Following [15-17] we have

$$\mathrm{var}(\hat{\theta})\geq-\frac{1}{E\{\partial^2\ln f_{\vec{x};\theta}(\vec{x};\theta)/\partial\theta^2\}} \tag{4}$$

The function $\ln f_{\vec{x};\theta}(\vec{x};\theta)$ is well known as the *"log likelihood"* function of θ (LLF). Its maximum likelihood estimate can be obtained from the equation:

$$\frac{\partial\ln f_{\vec{x};\theta}(\vec{x};\theta)}{\partial\theta}=0 \tag{5}$$

The first order of differential log likelihood function with respect to θ is called the maximum likelihood (ML) estimate. If the efficient estimate does not exist, then the ML estimate will not achieve the lower bound and hence it is difficult to ascertain how closely the variance of any estimate will approach the bound.

It is noted that the value of about 1.5 is strongly recommended if there is no information about α due to the 2nd order simulations of the LLF for an alpha-stable [17].

3. WAVE-BASED BAYESIAN ESTIMATOR

If we take the probability density of θ as $p(\theta)$; and the posterior density function as $f(\theta\mid x_1,\ldots,x_n)$, then the updated probability density function of θ is as follows:

$$f(\theta\mid x_1,\ldots,x_n)=\frac{f(\theta,x_1,\ldots,x_n)}{f(x_1,\ldots,x_n)}=\frac{p(\theta)f(x_1,\ldots,x_n\mid\theta)}{\int f(x_1,\ldots,x_n\mid\theta)p(\theta)d\theta} \tag{6}$$

If we estimate the parameters of the prior distributions of the signal s and noise q components of the wavelet coefficients c, we may use the parameters to form the prior PDFs of $P_s(s)$ and $P_q(q)$, hence the input/output relationship can be established by the Bayesian estimator, namely, let input/output of the Bayesian estimator = BE, we have:

$$BE=\frac{\int P_q(q)P_s(s)s\,ds}{\int P_q(q)P_s(s)ds} \tag{7}$$

$P_s(s)$ is the prior PDF of the signal component of the wavelet coefficients of the ultrasound image and $P_q(q)$ is the PDF of the wavelet coefficients corresponding to the noise.

In order to be able to construct the Bayesian processor in (7), we must estimate the parameters of the prior distributions of the signal (s) and noise (q) components of the wavelet coefficients (d). Then, we use the parameters to obtain the two prior PDFs $P_q(q)$ and $P_s(s)$ and the nonlinear input-output relationship BE.

Figure 1 shows the simulation results of input/output of BE with different α values for given γ (=25) and the mean of Poisson distribution (=35). It clearly shows that, for the given case, the curves with α = 0.1, 1.5, and 1.9 approximately correspond to the "hard", "soft", and "semisoft" functions respectively when compared with results in [8,18].

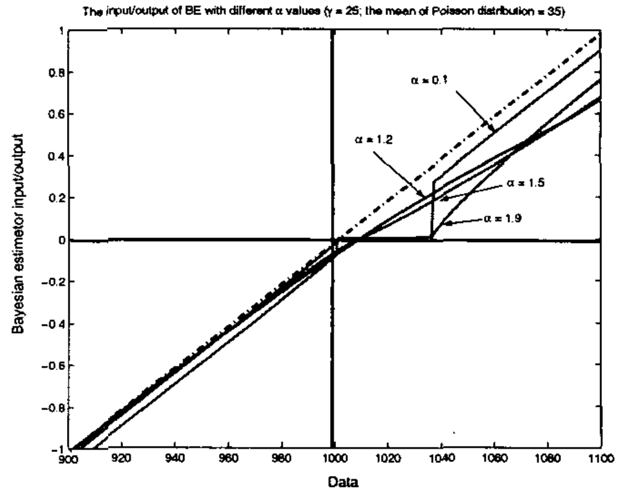

Figure 1: The input/output of BE with different α values (γ = 25; the mean of Poisson distribution = 35).

Unlike the case contaminated by Gaussian noise [15-18], the mean of Poisson noise plays a role in a BE as shown in Figure 2, where the parameters are the same as that in Figure 1 except for the mean of Poisson distribution is equal to 10 rather than 35

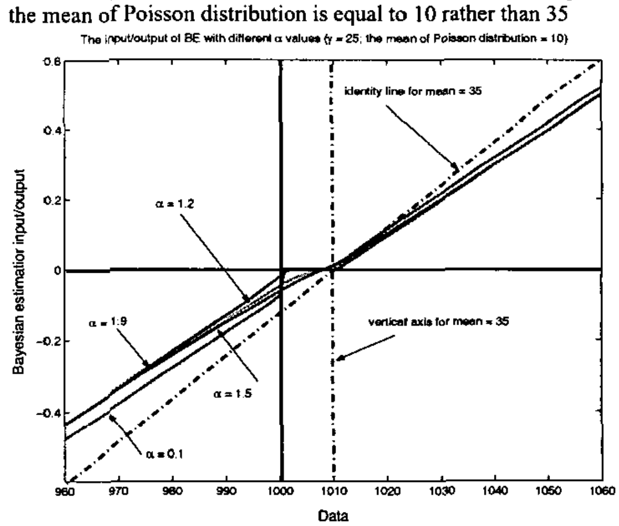

Figure 2: The mean of Poisson distribution affects BE. Here all parameters are the same as that in Figure 1 except the mean of Poisson distribution, which is 10.

It is clear that the curves with different α values are not significantly different and approximate a single function. This is as expected due to the relation, for Poisson distribution, between the variance and the mean. Taking the results from [15-17], we investigated how the ratio of γ/mean affects *BE* as shown in Figures 3 and 4. The means of Poisson distribution in Figures 3 and 4 are 35 and 10 respectively. Again, they confirm the fact that the mean of Poisson distribution plays a role in *BE*. In Figure 4, as in Figure 2, we show the corresponding vertical axis and "identity line" for previous mean. They also show that γ will affect the output of *BE*, since $\gamma \in \mathbf{R}$ is the dispersion of the distribution. The ratio of 30 and 20 correspond to "soft" and "semi-soft" functions respectively.

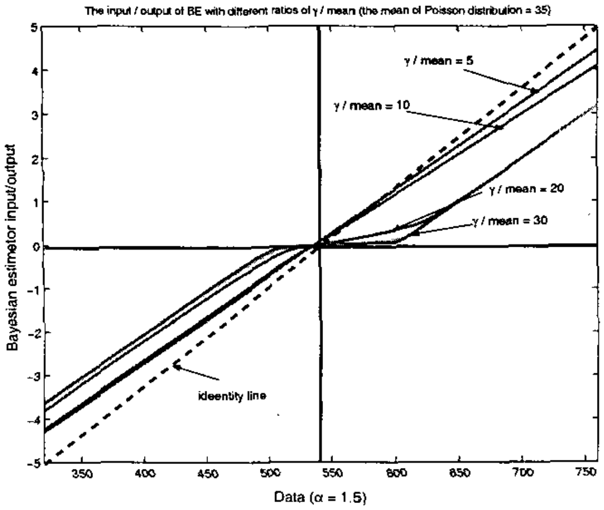

Figure 3: The input/output of BE with different ratios of γ/mean (with the mean of Poisson mean = 35, α = 1.5).

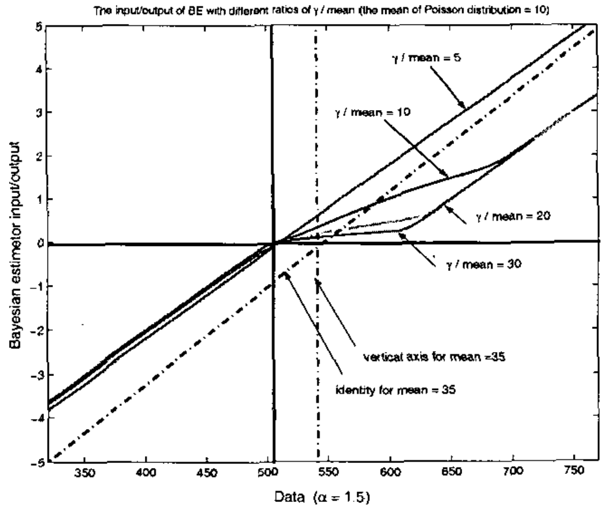

Figure 4: The input/output of BE with different ratios of γ/mean (with the mean of Poisson mean = 10, α = 1.5).

4. SOME EXAMPLES

When we make measurements, we have no information about the noise value of the image we obtained. The only information one may have is from experience in judgement of the noise level, which becomes the outline of the denoising strategy. We take the parameters α=1.5, γ/mean = 20, in the two cases of *BE* with the mean of Poisson distributions equal to 35 (Figure 8) and 10 (Figure 9). In order to compare, we show the original image called "dust storm" taken in NSW, Australia in Figure 5, together with a copy contaminated by Gaussian noise in Figure 6 and a copy contaminated by Poisson noise in Figure 7. Note the differences between the two noisy images. In particular that for the Poisson contaminated there is more noise in the brighter parts of the image. The Harr mother wavelet was used for this example. The output of the denoised image from *BE* is shown in Figure 8, where the Poisson distribution is 10 and the image with all the same conditions but changing the mean of the Poisson distribution to 35 is shown in Figure 9.

Figure 5: An Image of the "dust storm" in NSW Australia

Figure 6: The contaminated Figure 5 by Gaussian noise

Figure 7: Figure 5 contaminated by Poisson noise.

Comparisons of other denoising results are in table 1.

Method	1	2	3	4	5
S/MSE	14.21	14.30	14.01	14.87	14.68

Table 1: Comparison of denoising results with *BE* in signal to mean square error (*S/MSE*) in dB. Here 1= soft thresholding; 2 = Hard thresholding; 3 = Homomorphic Wiener; 4 = *BE* (mean =10), 5 = *BE* (mean =35).

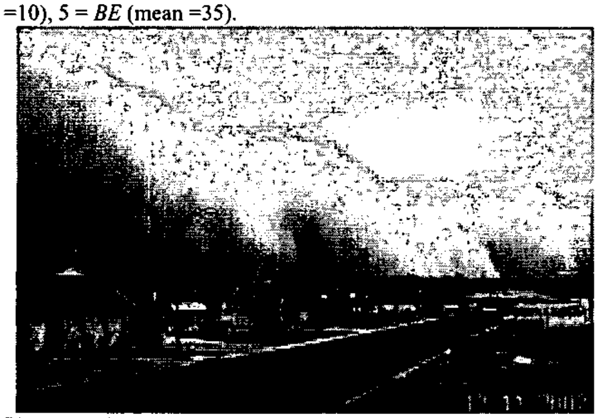

Figure 8: The denoised image from the designed *BE* (the mean of the Poisson distribution is 10).

Figure 9: The denoised image from the designed *BE* (the mean of the Poisson distribution is 35).

5. CONCLUSION

The technique using the wavelet-based Bayesian estimator has been extended to signal-dependent noise obeying the Poisson distribution. The statistician's Bayesian estimator theory is used not only to simplify the selection of parameters but also in some situations to provide more precise images than other methods.

6. ACKNOWLEDGEMENT

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