DESIGNING CUSTOMIZED HIERARCHICAL FUZZY LOGIC SYSTEMS
FOR MODELLING AND PREDICTION

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ABSTRACT
In this paper the design and development of a hierarchical fuzzy logic Systems are investigated. A new method using genetic algorithms for design of hierarchical fuzzy logic systems are proposed. This research study is unique in the way proposed method is applied to design and development of hierarchical fuzzy logic systems. The proposed method is then applied to financial modelling and prediction. A hierarchical fuzzy logic system is developed to predict quarterly interest rates in Australia. The new method proposed determines the number of layer in a hierarchical fuzzy logic system. The advantages and disadvantages of using hierarchical fuzzy logic systems for financial modelling is also considered. Good prediction of quarterly interest rate in Australia is obtained using the above method. The number of fuzzy rule used are reduced dramatically and prediction of interest rate is improved.

1. INTRODUCTION
Traditionally the predicting and modelling of uncertain dynamic systems, such as predicting of interest rates, has relied on complex mathematical models to describe the dynamic system to be modelled. These models work well provided the system meets the requirement and assumption of synthesis techniques. However due to uncertainty or sheer complexity of these systems, they are difficult to model and not easily adaptable to changes in the system which they were not designed for [1, 2, 5].

Computational Intelligence techniques such as Fuzzy Logic, Genetic Algorithms (GAs) and Neural Networks have been successfully used in the place of the complex mathematical systems [2, 4]. Fuzzy logic is an active research area [1, 2, 3, 4, 5]. Fuzzy modelling or fuzzy identification has numerous practical applications in control, prediction and inference. It has been found useful when the system is either difficult to predict and or difficult to model by conventional methods. Fuzzy set theory provides a means for representing uncertainties. The underlying power of fuzzy logic is its ability to represent imprecise values in an understandable form. The majority of fuzzy logic systems to date have been static and based upon knowledge derived from imprecise heuristic knowledge of experienced operators, and where applicable also upon physical laws that governs the dynamics of the process.

Although its application to industrial problems has often produced results superior to classical control [4, 6], the design procedures are limited by the heuristic rules of the system. It is simply assumed that the rules for the system are readily available or can be obtained. This implicit assumption limits the application of fuzzy logic to the cases of the system with a few parameters. The number of parameters of a system could be large. The number of fuzzy rules of a system is directly dependant on these parameters. As the number of parameters increase, the number of fuzzy rules of the system grows exponentially [5, 7].

In fuzzy logic systems, there is a direct relationship between the number of fuzzy sets of input parameters of the system and the size of the Fuzzy Knowledge Base (FKB). Kosko and Isaka [16] call this the “Curse of Dimensionality”. The “curse” in this instance is that there is exponential growth in the size of the FKB.

\[ k = m^n \] (1)

where \( k \) is the number of rules in the FKB, \( m \) is the number of fuzzy sets for each input and \( n \) is the number of inputs into the fuzzy system.

As the number of fuzzy sets of input parameters increase, the number of rules increases exponentially. There are a number of ways that this exponential growth in the size of the FKB can be contained. The most obvious is to limit the number of inputs that the system is using. However, this may reduce the accuracy of the system, and in many cases, render the system being modelled unusable. Another approach is to reduce the number of fuzzy sets that each input has. Again, this may reduce the accuracy of the system [2]. The number of rules in the FKB can also be trimmed if it is known that some rules are never used. This can be a time-consuming and tedious task, as every rule in the FKB may need to be looked at.

Raju and Zhou [7] suggested using a Hierarchical Fuzzy Logic structure for such fuzzy logic systems to overcome this problem. By using hierarchical fuzzy logic systems the number of fuzzy rules in the system are reduced thereby reducing the computational time
while maintaining the systems robustness and efficiency.
In this paper the design and development of a
hierarchical fuzzy logic systems using genetic
algorithms to model and predict interest rate in
Australia is considered. Genetic algorithms are
employed as an adaptive method for design and
development of hierarchical fuzzy logic systems.

2. HIERARCHICAL FUZZY LOGIC SYSTEMS

The Hierarchical Fuzzy Logic structure is formed by
having the most influential inputs as the system variables
in the first level of the hierarchy, the next
important inputs in the second layer, and so on. If the
hierarchical fuzzy logic structure contains \( n \) system
input parameters and \( L \) number of hierarchical levels
with \( n_i \) the number of variables contained in the \( i \)th
level, the total number of rules \( k \) is then determined by:

\[
k = \sum_{i=1}^{L} n_i^L
\]

where \( m \) is the number of fuzzy sets. The above
equation means that by using a hierarchical fuzzy logic
structure, the number of fuzzy rules for the system is
reduced to a linear function of the number of system
variables \( n \), instead of an exponential function of \( n \) as is
the conventional case [7]. The first level of the
hierarchy gives an approximate output, which is then
modified by the second level rule set, and so on. This is
repeated for all succeeding levels of the hierarchy. One
problem occurs when it is not known which inputs to
the system have more influence than the others. This is
the case in many problems. In some case statistical
analysis could be performed on the inputs to determine
which ones have more bearing on the system.

3. INTEGRATED HIERARCHICAL FUZZY
LOGIC AND GENETIC ALGORITHMS

Genetic Algorithms (GAs) [8, 9], are powerful search
algorithms based on the mechanism of natural selection
and use operations of reproduction, crossover, and
mutation on a population of strings. A set (population)
of possible solutions, in this case, a coding of the fuzzy
rules of a fuzzy logic system, represented as a string of
numbers. New strings are produced every generation by
the repetition of a two-step cycle. First each individual
string is decoded and its ability to solve the problem is
assessed. Each string is assigned a fitness value,
depending on how well it performed. In the second stage
the fittest strings are preferentially chosen for
recombination to form the next generation.
Recombination involves the selection of two strings,
the choice of a crossover point in the string, and the
switching of the segments to the right of this point,
between the two strings (the cross-over operation).
Figure 1 shows the combination of fuzzy logic and
genetic algorithms for generating fuzzy rules.

For encoding and decoding of the fuzzy rule for a fuzzy
logic system, first the input parameters of the fuzzy logic
system is divided into fuzzy sets. Assume that the fuzzy
logic system has two inputs \( \alpha \) and \( \beta \) and a single output
\( \delta \). Assume also that the inputs and output of the system
is divided into 5 fuzzy sets. Therefore a maximum of
twenty five fuzzy rules can be written for the fuzzy logic
system.
The consequent for each fuzzy rule is determined by
 genetic evolution. In order to do so, the output fuzzy
sets are encoded. It is not necessary to encode the input
fuzzy sets because the input fuzzy sets are static and do
not change.

![Figure 1. combination of fuzzy logic and genetic algorithms
for fuzzy rule generation.](image)

The fuzzy rules relating the input variables (\( \alpha \) and \( \beta \)) to
the output variable (\( \delta \)) have twenty five possible
combinations. The consequent of each fuzzy rule can be
any one of the five output fuzzy sets. Assume that the
output fuzzy sets are: \( \text{NB} \) (Negative Big), \( \text{NS} \) (Negative
Small), \( \text{ZE} \) (Zero), \( \text{PS} \) (Positive Small), and \( \text{PB} \)
(Positive Big). Then the output fuzzy sets are encoded
by assigning \( 1 = \text{NB} \) (Negative Big), \( 2 = \text{NS} \) (Negative
Small), \( 3 = \text{ZE} \) (Zero), \( 4 = \text{PS} \) (Positive Small), and \( 5 = \text{PB} \)
(Positive Big). Genetic algorithms randomly encode
each output fuzzy set into a number ranging from 1 to 5
for all possible combinations of the input fuzzy
variables. A string encoded this way can be represented
as:

<table>
<thead>
<tr>
<th>4</th>
<th>3</th>
<th>5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Each individual string is then decoded into the output
linguistic terms. The set of fuzzy rules thus developed,
is evaluated by the fuzzy logic system based upon a
fitness value which is specific to the system. At the end
of each generation, (two or more) copies of the best
performing string from the parent generation is included
in the next generation to ensure that the best performing
strings are not lost. Genetic algorithms then performs
the process of selection, crossover and mutation on the
rest of the individual strings. Selection and crossover
are the same as a simple genetic algorithms [9] while
the mutation operation is modified.
Crossover and mutation take place based on the
probability of crossover and mutation respectively.
Mutation operator is changed to suit this problem. For

mutation, an allele is selected at random and it is replaced by a random number ranging from 1 to 5. The process of selection, crossover and mutation are repeated for a number of generations till a satisfactory fuzzy rule base is obtained. We define a satisfactory rule base as one whose fitness value differs from the desired output of the system by a very small value.

4. HIERARCHICAL FUZZY LOGIC SYSTEM FOR INTEREST RATE PREDICTION

There is a large interest by investors and government departments in the ability to predict future interest rate fluctuations from current economic data. Economists, and investors, have been unable to find all the factors that influence interest rate fluctuations. It is agreed however that there are some major economic indicators released by the government [12] that are commonly used to look at the current position of the economy. These indicators used in this paper \[10, 11\] are as follows:

- **Interest Rate** which is the indicator being predicted. The Interest Rate used here is the Australian Commonwealth government 10-year treasury bonds.
- **Job Vacancies** is where a position is available for immediate filling or for which recruitment action has been taken.
- **The Unemployment Rate** is the percentage of the labour force actively looking for work in the country.
- **Gross Domestic Product** is an average aggregate measure of the value of economic production in a given period.
- **The Consumer Price Index** is a general indicator of the rate of change in prices paid by consumers for goods and services.
- **Household Saving Ratio** is the ratio of household income saved to households disposable income.
- **Home Loans** measure the supply of finance for home loans, not the demand for housing.
- **Average Weekly Earnings** is the average amount of wages that a full time worker takes home before any taxes.
- **Current Account** is the sum of the balances on merchandise trade, services trade, income and unrequited transfers.
- **Trade Weighted Index** measures changes in our currency relative to the currencies of our main trading partners.
- **RBA Commodity Price Index** provides an early indication of trends in Australia’s export Prices.
- **All Industrial Index** provides an indication of price movements on the Australian Stock Market.
- **Company Profits** are defined as net operating profits or losses before income tax.
- **New Motor Vehicles** is the number of new vehicles registered in Australia.

By creating a system that contained all these indicators, we would be in a much better position to predict the fluctuations in interest rates. A fuzzy logic system that used every indicator and had five fuzzy sets for every indicator would result in a large FKB consisting of over six billion rules! As can be imagined, this would require large computing power to not only train the fuzzy logic system with a genetic algorithm, but also large storage and run-time costs when the system is operational. Even if a computer could adequately handle this large amount of data, there is still the problem in having enough data to properly train every possible rule. To overcome this problem a hierarchical fuzzy logic structure for the fuzzy logic system can be constructed. By using a hierarchical fuzzy logic system, the number of fuzzy rules of the system is reduced hence computational times are decreased resulting in a more efficient system. A novel way to tackle this problem would be to group the relevant indicators and to build a fuzzy knowledge base for each group. The first step is to divide the indicators into smaller-related groups. This problem was investigated in [10, 11] and is shown below:

1. **Employment** (Job Vacancies, Unemployment Rate)
2. **Country** (Gross Domestic Product, Consumer Price Index)
3. **Savings** (Household Saving Ratio, Home Loans, Average Weekly Earnings)
4. **Foreign** (Current Account, RBA Index, Trade Weighted Index)
5. **Company** (All Industrial Index, Company Profit, New Motor Vehicles)

The Interest Rate is included with each of the groups above. To learn the fuzzy knowledge base for each group, a genetic algorithm was implemented. The genetic algorithms had a population size of 500 with a crossover rate of 0.6 and a mutation rate of 0.01 and it was run for 10000 generations over 10 years (a period of 40 quarters) data. Fitness of each string of the genetic algorithm was calculated as the sum of the absolute differences from the predicted quarter and the actual quarters interest rate. The fitness was subtracted from an ‘optimal’ fitness amount, which was decided to be 30 as it was unlikely the error amount would be higher than this over 10 years \[10, 11\]. The fitness of the system is calculated by the following formula:

\[
\text{fitness} = 30 - \sum_{i=0}^{30} \text{abs}(P_{i} - I_{i})
\]

An Elitist strategy was used in that the best population generated was saved and entered in the next generation (two copies of the string with best fitness was included to the next generation).

The five fuzzy knowledge bases created form the top layer of the hierarchy are shown in Figure 2 \[10, 11\]. Kingham et al \[10, 11\] designed and connected together the fuzzy knowledge bases to form a final fuzzy knowledge base system. The final fuzzy knowledge base system shown in Figure 2 then uses the predicted interest rate from the five above groups to produce a final interest rate prediction. The number of fuzzy rules for each group is shown in Figure 2 \[10, 11\].
The final hierarchical FKB contains 3125 rules [10, 11] giving the total number of rules learnt as 5250. This is a significant reduction from the 6 billion rules that would have been used previously. This allows quicker training time without the need for huge computer resources [10, 11]. Good prediction of Australian quarterly Interest rate can be obtained using the above system. The number of fuzzy rule are used are also reduced dramatically.

**Level 2**

<table>
<thead>
<tr>
<th>Employment</th>
<th>Country</th>
<th>Savings</th>
<th>Foreign</th>
<th>Company</th>
</tr>
</thead>
<tbody>
<tr>
<td>125 rules</td>
<td>125 rules</td>
<td>625 rules</td>
<td>625 rules</td>
<td>625 rules</td>
</tr>
</tbody>
</table>

**Predicted Quarterly Interest rate**

**Figure 2.** Hierarchical Knowledge Base Flow [10, 11]

However there is still a question: Does a two layer hierarchical architecture provides the best solution? To answer this question, one can start building three, four layer hierarchical fuzzy logic system by trial and error to possibly find the correct number of layers required. This could be cumbersome problem. We need to know how many layers are required and which fuzzy knowledge base should be used in each layer. Genetic algorithms can be used to solve this problem by determining the number of layer in the hierarchical fuzzy logic system and the correct combination of FKBs for each layer see Figure 3.

**Layer 1**

<table>
<thead>
<tr>
<th>Company</th>
<th>Savings</th>
<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td>625 rules</td>
<td>625 rules</td>
<td>625 rules</td>
</tr>
</tbody>
</table>

**Layer 2**

- Employment | Country |
- 625 rules | 625 rules |

**Predicted Quarterly Interest rate**

**Figure 3.** A three-layer hierarchical fuzzy logic system – 3125 fuzzy rules

Next the performance of genetic algorithms for design and development of hierarchical fuzzy logic systems is considered. The system is developed in such a way to provide the possible best architecture for designing hierarchical fuzzy logic systems for prediction of Interest Rate in Australia. Using the economic indicators five fuzzy logic systems were developed from five groups each produce a predicted interest rate for the next quarter. Genetic algorithms were then used to design and develop a hierarchical fuzzy logic system.

The hierarchical fuzzy logic system developed was then used to predicted interest rate For each of these groups, the current quarter’s interest rate is included in the indicators used [10, 11].

The advantage of using this hierarchical fuzzy logic structure is that the number of rules used in the FKB’s has been reduced substantially. For encoding and decoding of the hierarchical fuzzy logic system, first a number is allocated to each fuzzy logic system developed from group of indicators. For this simulation the number allocated to each group is shown below:

1 = Employment, 2 = Country, 3 = Savings, 4 = Foreign, 5 = Company

The number of layers and the fuzzy logic system/s for each layer is determined by genetic algorithms. In order to do so a number is allocated to each fuzzy logic system. Genetic algorithms randomly encode each fuzzy logic system into a number ranging from 1 to 5 for all possible combinations of the fuzzy logic systems. The level in the hierarchy in which a fuzzy logic system is allocated to, is also encoded each string. A string is encoded this way can be represented as:

Figure 3 above. The set of hierarchical fuzzy logic systems thus developed, are evaluated and a fitness value is given to each string. At the end of each generation, (two or more) copies of the best performing string from the parent generation is included in the next generation to ensure that the best performing strings are not lost. Genetic algorithms then performs the process of selection, crossover and mutation on the rest of the individual strings. Crossover and mutation take place based on the probability of crossover and mutation respectively. Mutation operator is changed to suit this problem. The process of selection, crossover and mutation are repeated for a number of generations till a satisfactory hierarchical fuzzy logic system is obtained.

We define a satisfactory hierarchical fuzzy logic system as one whose fitness value (predicted interest rate) differs from the desired output of the system (in this case the actual interest rate) by a very small value.

We calculate the average error of the system for the training set and tests sets using the following formula [10, 11]:

$$E = \frac{1}{n} \sum_{i=1}^{n} \text{abs}(P_i - A_i)$$  \hspace{1cm} (4)
where E is the average error, \( P_i \) is the Predicted interest rate at time period \( i \), \( A_i \) is the actual interest rate for the quarter and \( n \) is the number of quarters predicted. By using genetic algorithms to design and develop hierarchical fuzzy logic system better results were obtained. The hierarchical fuzzy logic systems developed using genetic algorithms perform predict the interest rate to different degree of accuracy. It is however interesting to see that genetic algorithms is capable of providing different hierarchical fuzzy logic system for predicting the interest rate. It is now possible to choose the best hierarchical fuzzy logic system among those suggested by genetic algorithms.

The result of the top performing five hierarchical fuzzy logic systems designed by genetic algorithms is given in Table 1. Comparison of average errors of these five best hierarchical fuzzy logic systems designed and developed using genetic algorithms is also shown in Table 1.

<table>
<thead>
<tr>
<th>HFL #</th>
<th>Training Error</th>
<th>Testing Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>HFL #1</td>
<td>0.356</td>
<td>0.659</td>
</tr>
<tr>
<td>HFL #2</td>
<td>0.343</td>
<td>0.663</td>
</tr>
<tr>
<td>HFL #3</td>
<td>0.289</td>
<td>0.494</td>
</tr>
<tr>
<td>HFL #4</td>
<td>0.274</td>
<td>0.441</td>
</tr>
<tr>
<td>HFL #5</td>
<td>0.291</td>
<td>0.398</td>
</tr>
</tbody>
</table>

Table 1. Comparison of Average Errors of hierarchical fuzzy logic (HFL) systems designed and developed using GA

5. CONCLUSION AND FURTHER INVESTIGATIONS

In this paper an innovative method is used to design and develop hierarchical fuzzy logic systems. Genetic algorithms is used as an adaptive learning method to design a hierarchical fuzzy logic systems to predict the fluctuations of the 10-year Australian treasury bond using Australian economic data. Using hierarchical fuzzy logic systems, the number of fuzzy rules in the fuzzy knowledge base is reduced dramatically hence computational times are decreased resulting in a more efficient system.

The application of the proposed method to modelling and prediction of interest rate using Australian economic indicators is considered. Genetic algorithms are also used to obtain the fuzzy rules for each fuzzy logic system of a hierarchical fuzzy logic system.

From simulation results it was found that the hierarchical fuzzy logic system is capable of making accurate predictions of the following quarter’s interest rate. The hierarchical fuzzy logic systems used a fuzzy knowledge base which contains all the rules of the system, this allows an expert to inspect and make any modifications if necessary.

The research work performed in this paper is unique in the way the hierarchical fuzzy logic systems are developed. The application of this method to several industrial problems such as robotic control and collision avoidance of multi-robot systems is currently under consideration.

REFERENCES


