Recombination noise in semiconductor junction devices

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Abstract: The evolution of conceptual models of recombination noise generation in bipolar semiconductor junctions is explored with particular reference to recent developments in non-classical light generation. This development is traced from the early work pioneered by van der Ziel through to recent work on sub-Poissonian light generation initiated by Yamamoto. This recent work has emphasised the importance of the driving impedance in suppressing recombination noise. It has helped to resolve several longstanding ambiguities and misunderstandings concerning the fundamentals of shot noise generation in laser diodes and light-emitting diodes, as well as in bipolar junction diodes and transistors, and allows a common conceptual approach to shot noise generation and propagation in photonic and electronic devices. Surprisingly, it also lends support to early suggestions by van der Ziel, subsequently regarded as erroneous by Buckingham and Faulkner, that bipolar junction shot noise does in fact originate in the transport of minority carriers across the depletion region of macroscopic junctions, although only in the limit of low injection.

1 Introduction

The invention of the semiconductor laser and the light-emitting diode, in which radiative recombination dominates the recombination process, provides a diagnostic tool that enables the details of the internal (radiative) recombination process to be accessed in the form of the emitted recombination light. Previous models of the internal noise processes in semiconductor junction devices could not be directly tested. Instead, noise measurements and the associated noise equivalent circuits were restricted to the external terminal current noise and the terminal voltage noise. The noise generators used in the noise equivalent circuits were required only to reproduce the current and voltage noise external to the device. One of the consequences of this was that early diode and transistor noise models [1–5] did not have as firm a physical basis as is now possible. In fact, as subsequently pointed out by Buckingham, Faulkner and Robinson [6–8], early shot noise models were inconsistent with the Shockley model of the bipolar junction, despite their success in predicting external current noise. Another consequence has been the emergence of noise equivalent circuits in which the current and voltage noise generators, introduced for reasons of analytic convenience, are sometimes mistakenly assigned a physical reality although they do not necessarily represent independent fundamental physical processes [5, 9].

In this paper, as in a previous presentation [10], the term ‘recombination current noise’ is used to refer to fluctuations in the radiative recombination rate that would be observed as photocurrent fluctuations in an ideal 100% efficient photon detector, capable of collecting and detecting all of the photons emitted from a light-emitting junction. Since every electron–hole recombination event in an ideal junction then maps to a corresponding electron–hole generation event in such an ideal photon detector, the statistical point process that describes the recombination events is accessible on an event-by-event basis to an efficient, high-resolution photon counter. In practice, such a counter will not detect all events because its counting efficiency will be less than unity. This restriction is not usually a severe one since the random deletion of events from the output of such a non-ideal detector introduces additive partition noise and is easily handled as a Bernoulli process. This permits an accurate description of the original process to be recovered for reasonable values of the counting efficiency. In practice it is not necessary to resolve individual recombination events in order to obtain a useful statistical description of the recombination process. Instead, broadband spectral and time domain measurements of photodetector current fluctuations serve equally well to characterise the recombination rate fluctuations and hence illuminate the basic processes. There is of course an implicit assumption that the results and analysis of such observations can be applied with appropriate caveats to other bipolar junction devices in which nonradiative processes dominate and which are therefore inaccessible to photoelectric examination. It might also be argued that photoelectric noise measurements merely replace measurements on one pair of terminals (the junction diode) with those on another pair (the photodetector) and are therefore no more direct. However, since photodetectors are electrically isolated from the generating light-emitting junctions and are well understood in terms of quantum detection theory, such issues as their impedance loading are irrelevant and do not, for example, influence the amount of photonic shot noise observed.

As will be shown, the recombination current noise measured in this way turns out to be strongly influenced by the impedance of the bias circuit configuration, although it obviously has its physical origin in the carrier generation, transport and recombination processes in the vicinity of the junction. Its measurement has provided significant new

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insights into semiconductor junction dynamics as well as foreshadowing new quantum information technologies and applications based on the suppression of photonic shot noise. Indeed, one of the unexpected outcomes of photoelectric measurements of recombination current noise in semiconductor light-emitting junctions was the discovery that the internal recombination rate fluctuations did not necessarily reach the full shot noise level and that the corresponding recombination events were 'anti-bunched' with sub-Poissonian variance. The discovery of 'quiet' light originating in recombination current noise fluctuations below the full shot noise level in light-emitting semiconductor heterojunction diodes [11–14] followed shortly after the first measurements of sub-Poissonian light from cooled semiconductor diode lasers by the Yamamoto group at NTT [15]. However, unlike the case of the laser diode, a quantum mechanical analysis of the generation process [16, 17] is not necessary and semiclassical statistical methods are adequate to analyse the internal recombination dynamics of light-emitting diodes [18].

The concepts, mechanisms and the techniques for suppressing the shot noise associated with the recombination process in semiconductor junction devices, as revealed by measurements of intensity fluctuations in the emitted light, has attracted wide interest because of the potential applications of quiet light and controlled photon emission technologies in metrology, telecommunications, computing and cryptography [19]. An authoritative review of non-classical (quantum theory-based) light emission from light-emitting diodes and laser diodes is given in the monograph published by Kim, Somani and Yamamoto [20].

The aim is to describe the phenomenology of the shot noise processes that take place in semiconductor junction devices as revealed historically, first by external current measurements and more recently by photoelectric measurements. Analytic discussion will be simplified by restricting the discussion to diodes in which the terminal current is due entirely to recombination as shown in Fig. 1. This is not a significant restriction to gaining information on the basic shot noise generating processes, the thermal transport and recombination of charge carriers, in junction diodes and 'intrinsic' transistors.

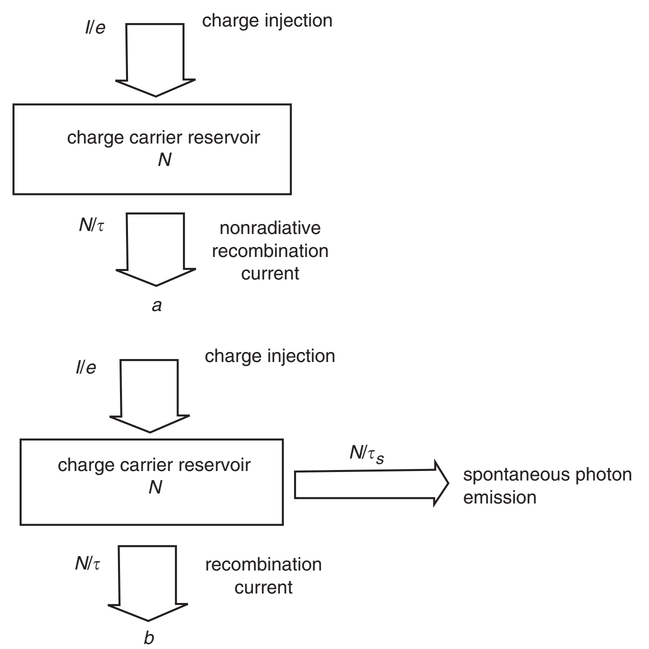

Fig. 1 *Models of an ideal semiconductor bipolar junction diode (Edwards [24])*
a Ideal light-emitting bipolar junction diode
b Showing the basic noise generating processes of charge injection, storage and recombination

2 Semiconductor junction shot noise

2.1 Noise measurements

Two different noise currents and one noise voltage are directly accessible to measurement. These are:

(i) terminal current noise: the noise in the current flowing into the terminals of the device (measured with a low impedance probe such as a transimpedance amplifier);

(ii) junction terminal voltage noise: the noise in the voltage drop measured across the terminals of the device (measured with a high impedance probe); and

(iii) electron–hole recombination noise as deduced from photoelectric measurements made on the emitted light with a photoemissive, photoconductive or semiconductor junction diode detector operating in either photon-counting, photocurrent or photovoltaic modes.

In the following simplified discussion we assume either a long-base homojunction diode, that is a diode in which the width of the bulk material recombination layer is much longer than the minority carrier diffusion length, or alternatively a heterojunction diode in which injection is restricted to the active region. These assumptions simplify the discussion considerably since minority carriers cannot then diffuse as far as the ohmic electrodes, and all of the terminal current must then be due to recombination as assumed in the well known equivalent circuits of Figs. 2*a* and 2*b*. For simplicity the ohmic substrate resistance has been assumed to be negligible.

2.2 Terminal current noise

The charge carrier number fluctuations in the vicinity of a macroscopic semiconductor junction $N(t) = N + n(t)$ originate in the processes of charge injection, transport and recombination. Figure 1 illustrates the net effect of these processes, summarised in the following rate equation [18]:

$$dN(t)/dt = I(t)/e - N(t)/\tau + f_n(t) \qquad (1)$$

The first term on the right-hand side, $I(t) = I + i(t)$, represents the current supplied from an external circuit. It is taken to equal the net (fluctuating) rate at which charge is

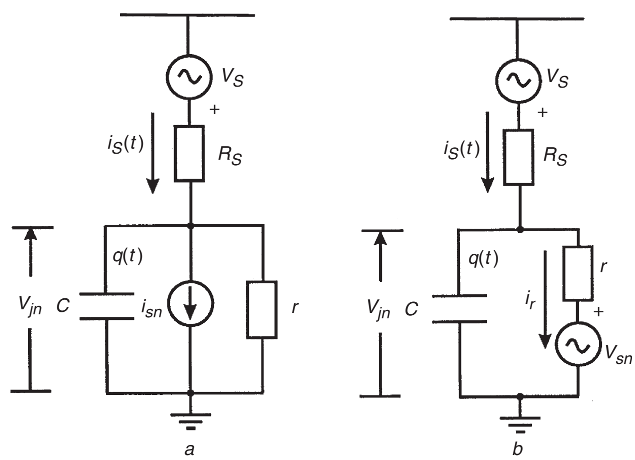

Fig. 2 *Generic noise equivalent circuits of ideal semiconductor junction diode with differential resistance r and junction capacitance C, biased through external resistance R_s (Edwards [19])*
a Shot noise current generator i_{sn}
b Equivalent shot noise voltage generator $v_{sn} = -i_{sn}r$

injected across the depletion layer at the junction into the 'active' (recombination) region. The second term in (1) in which τ is the mean lifetime of the diffusing carriers, comprises the mean recombination rate N/τ plus a noise term $n(t)/\tau$. The third term is the Langevin noise term $f_n(t)$ representing the shot noise generator [17]. This is the stochastic driving term. In this model the physical basis of this noise term is the random electron–hole recombination process; charge carriers are assumed to recombine with constant mean lifetime τ. For a fixed population of carriers the recombination then follows Poissonian statistics and the recombination rate exhibits full shot noise fluctuations.

Linearisation of (1) yields

$$dn(t)/dt = i_s(t)/e - n(t)/\tau + f_n(t) \tag{2}$$

Referring to the noise equivalent circuits of Fig. 2, corresponding state equations can be written down in terms of the state variables. These are the junction potential fluctuation $v_{jn}(t)$, the injected current $i(t)$ and the charge $q(t)$ stored in the junction capacitance, which for heavy bias current is just the so called 'diffusion capacitance' C_d. This gives

$$\begin{aligned} dq/dt &= i_s(t) - q(t)/rC + f_i(t) \\ &= i_s(t) - q(t)/rC - i_{sn}(t) \end{aligned} \tag{3}$$

and

$$\begin{aligned} Cdv_{jn}/dt &= i_s(t) - v_{jn}(t)/r - i_{sn}(t) \\ &= i_s(t) - v_{jn}(t)/r + v_{sn}(t)/r \end{aligned} \tag{4}$$

where $ef_n(t) = f_i(t) = -i_{sn}(t) = v_{sn}(t)/r$.

Referring to the corresponding noise equivalent circuits of Fig. 2, in which the stored charge fluctuation $q(t) = n(t)e = Cv_{jn}(t)$ and full shot noise voltage generator $v_{sn}(t) = -i_{sn}(t)r$ shows that in this model the junction voltage fluctuation is a direct measure of the carrier number fluctuation $n(t)$.

From Fig. 2, if the bias resistance $R_s \ll r$, the full recombination shot noise current $i_{sn}(t) = i_s(t)$ then evidently flows in the external circuit at low frequencies in agreement with circuit noise measurements. From (2)–(4) it is evident that the current noise in the external circuit, $i_{sn}(t) = -ef_n(t)$ is at the full shot noise level. Setting $R_s \ll r$ pins the junction voltage and fixes the stored electron population N, so that $dn(t)/dt = n(t) = 0$ and the charge carriers recombine randomly with fixed mean lifetime τ. This constitutes a Poisson point process with correlation function $\langle f_n(t) f_n(t') \rangle = 2N\delta(t-t')/\tau$, mean rate N/τ, mean square value $\langle f_n^2 \rangle = (N/\tau)\Delta f$ and double-sided spectral density equal to the mean recombination rate N/τ [19].

The low-frequency mean square terminal current spectral density is then, as expected

$$S_i(\omega) = 2\langle f_i^2 \rangle/\Delta f = 2e^2 \langle f_n^2 \rangle/\Delta f = 2Ie \tag{5}$$

If the bias resistance is raised, reference to either of the two equivalent circuits of Fig. 2 shows that the terminal current noise is reduced accordingly, again in agreement with circuit noise measurements. Nevertheless, since the current generator in Fig. 2*a* and the voltage generator in Fig. 2*b* are identified as recombination noise generators, the natural deduction is that the recombination noise level should remain unchanged [21]. Phototelectric measurements of the recombination current noise have shown this conclusion to be quite incorrect. In fact, contrary to expectations based on the behaviour of a constant voltage-driven LED, very large (>20 dB) reductions in the recombination noise level have been deduced from photoelectric measurements of the recombination light emitted from light-emitting diodes [11–14]. This has forced a major change in our understanding of the phenomenonology of both radiative and nonradiative recombination in semiconductor junctions, the subject of this review.

2.3 Recombination current noise

Referring again to (1) and (2), the second and third terms on the right-hand side together represent the fluctuating recombination rate, a population-dependent, time varying rate $(N(t)/\tau)$ plus an intrinsic stochastic (Poissonian) fluctuation $f_n(t)$. The former term represents the response of the reservoir population $N(t)$ and hence of the (population number-dependent) recombination rate to two sources of noise: (i) external source 'pump' noise $i_s(t)/e$, and (ii) the stochastic charge recombination process itself as represented by the third term, the Langevin noise term $f_n(t)$. A long-standing erroneous view held that recombination noise was represented solely by the latter (Langevin) term and therefore could not be reduced below full shot noise level, irrespective of the external loop impedance. Referring to (1) and (2), the recombination current fluctuation is the sum of the last two terms on the right-hand side

$$i_r(t) = -edn(t)/dt = en(t)/\tau - ef_n(t) \tag{6}$$

From Fig. 2*b*, the recombination current noise is then set by the difference between the Thevenin equivalent shot noise voltage $v_{sn} = -i_{sn}r$ and the terminal voltage fluctuation v_{jn}

$$= (v_{jn}(t) - v_{sn}(t))/r \tag{7}$$

This equivalent circuit then shows clearly that the full shot noise current flows only when the terminal voltage is pinned, that is when $v_{jn}(t) = 0$. This occurs at high frequencies and when $R_s \ll r$.

2.4 Recombination current noise suppression

The voltage source $v_s(t)$ in Fig. 2 represents the thermal Nyquist noise voltage associated with resistance R_s. The mean square pump noise current, $\langle i_s(t)^2 \rangle$ can be reduced to negligible proportions and the terminal voltage can be unpinned by raising the value of the resistance R_s. If the terminal voltage is allowed to freely fluctuate then $N(t)$ and $V_{jn}(t)$ will also fluctuate freely. If in addition the injection current noise is suppressed, then $i_r(t) = 0$.

This is the basis of the high impedance method of low-frequency recombination noise suppression in which both the pump noise and the recombination noise currents are suppressed. It has been interpreted [18] as a 'leaky reservoir' model of shot noise suppression in which the freely fluctuating reservoir level $(N(t))$ responds to the stochastic fluctuations in recombination rate $(f_n(t))$, and thus provides rate compensation (Teich and Saleh [22]) , through the term $(n(t)/\tau)$, by this negative feedback mechanism.

2.5 Recombination noise spectra

Taking Fourier transforms for the case $R_S \gg r$, the spectrum of the suppressed shot noise becomes

$$\begin{aligned} S_i(w) &= 2\omega^2\tau^2 < f_i^2 > /\Delta f(1 + \omega^2\tau^2) \\ &= 2Ie\omega^2\tau^2/(1 + \omega^2\tau^2) \end{aligned} \tag{8}$$

This has the character of single pole high-pass filtered shot noise and vanishes in the low-frequency limit of $\omega\tau \ll 1$. Thus, shot noise suppression at frequencies $\omega \ll 1/\tau = 1/rC$, is evidently achieved by making the external impedance R_s much greater than r, the internal differential resistance of the junction, so that $i(t) = (v_s(t) - v_{jn}(t))/R_s$ vanishes in that limit. This illustrates low-frequency shot noise suppression according to the fluctuating charge reservoir model. The electron reservoir number (voltage) fluctuation spectrum has a complementary lowpass character with a total mean square fluctuation of $\langle N \rangle/2$, just one half the Poissonian value. The noise suppression bandwidth is given by

$$B = 1/2\pi\tau = 1/2\pi r C_d \tag{9}$$

independently of the bias current since in the ideal Shockley model [23] the effective recombination life time is fixed. As pointed out in Section 2.8 this highly simplified model does not correctly predict the bandwidth over which noise suppression occurs, except at high injection levels.

2.6 Photoelectric measurements

The possibility that the photonic shot noise in the light from semiconductor lasers could be suppressed at the source was suggested in 1986 by Yamamoto *et al.* [16, 17]. This quantum mechanical analysis of 'amplitude squeezing' of the light from semiconductor lasers was followed in 1987 by observations of sub-Poissonian photonic noise by Machida *et al.* [15] in the light from strongly pumped cooled laser diodes and from light emitting diodes by Tapster *et al.* [11] and later by Edwards *et al.* [12–14], Kim *et al.* [20, 25], Yamanishi *et al.* [26, 27] and others.

The terms 'quiet', 'sub-Poissonian' and 'sub-shot noise' light are used interchangeably to refer to a photon beam in which the variance of the fluctuating photon count in a detector exposed to the beam is less than for the random (Poissonian) case, corresponding to an integrated mean square current fluctuation below the full shot noise level. A 'quiet' light beam is therefore visualised as a stream of photons of more or less constant number density over distances longer than some characteristic scale, giving rise to sub-Poissonian fluctuations in the photoelectron hole–pair generation rate in an ideal detector over corresponding integration times.

It was found that when laser and light-emitting diodes are operated with high impedance biasing circuits not only does this mode of operation suppress the external terminal current ('pump') fluctuation, but it also suppresses the internal fluctuation in the radiative recombination rate. The result is that the external circuit current noise and the electron–hole recombination current noise present in the emitted light are both suppressed below the full shot noise level. This result ran counter to the conventional wisdom that described the recombination process by Poissonian statistics, as would be expected were the stored charge to be held constant by application of constant voltage bias. For many years the analysis of laser diode noise was faulty in this respect. It was implicitly but erroneously assumed that the light from a strongly pumped laser diode would necessarily carry the full shot noise. As shown above this error arose as a consequence of a misinterpretation of the noise equivalent circuit and rate equations and neglected the internal negative feedback mechanisms that arise when the diode is operated with constant bias current.

The degree of shot noise suppression actually observed in the emitted light is in fact usually set by transmission losses and photon detection efficiency. The additive partition noise that arises in consequence of these losses ($\eta < 1$) can be easily calculated from

$$\langle i_p^2 \rangle = (1 - \eta)\langle i_{sn}^2 \rangle \tag{10}$$

where $\langle i_{sn}^2 \rangle$ is the full shot noise variance. The radiative recombination noise level in the junction can then be estimated with good precision unless the overall detection efficiency $\eta \ll 1$. Figure 3 represents this situation.

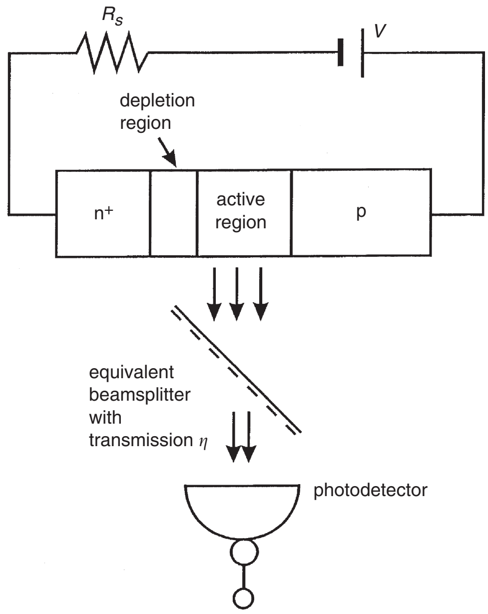

Fig. 3 *Optical measurement of recombination noise in a semiconductor light-emitting junction (Edwards [24])*
Losses and non-ideal detection are lumped together and represented by an optical beam splitter with transmission $\eta < 1$

The measured shot noise, relative to full shot noise, can then be written in the form

$$F_o = \eta F_i + (1 - \eta) \tag{11}$$

where F_o is the observed Fano factor (the noise variance relative to the Poissonian or full shot noise level) from which F_i , the Fano factor for the junction recombination noise, can be easily deduced. As predicted, measurements on the light from light-emitting diodes and laser diodes [26, 28–30] indicate a high degree high impedance current noise suppression as predicted, typically more than 20 dB, that is $F_i \leq 0.01$.

2.7 Photon-transport/bipolar junction transistor noise models

One outcome of the successful optical measurement and interpretation of recombination noise in semiconductor junctions has been the recognition of the close similarity between electronic shot noise generation and suppression in bipolar junction diodes and transistors on the one hand, and photonic shot noise in optocouplers and photon transport transistors [31] on the other. The original van der Ziel/Becker corpuscular theory was developed in the context of the successful theory of shot noise in thermionic vacuum diodes. Essentially it proposed that semiconductor shot noise originated in the stochastic transport of minority charge carriers across the depletion layer and in their subsequent random partitioning between base and collector. Although the physical basis for the theory was challenged by Buckingham [7] and Robinson [8] as being inconsistent with Shockley's diffusion model of a homojunction [23], it nevertheless provides an accurate empirical model of diode and bipolar transistor terminal current shot noise. It is not unreasonable to ask the question: why is this so? The answer is that the processes of shot noise generation in the base–emitter diode and partition noise generation between base and collector are generic processes that can be described without specific reference to the detailed microphysics. For example, Edwards *et al.* [30, 32, 33] recently proposed a 'neo-corpuscular' model of both electronic and photonic shot noise suppression based on this approach using the stochastic point process approach described by Teich and Saleh [22]. This accurately models sub-Poissonian circuit shot noise and recombination shot noise in junction diodes, bipolar junction transistors and photon-coupled transistors using the equivalent circuit and rate equation descriptions referred to earlier. This BJT/PTT noise model is based on the equivalent noise circuit of a reverse biased p-n junction photon detector (collector) coupled to a forward biased light emitting p-n junction (emitter) with positive current feedback applied from collector to base. A minority carrier/(photon) flux with embedded shot noise determined by the impedance in the base–emitter circuit is transported from the charge-carrier/(photon)-emitter to the charge-carrier/(photon)-collector, subject to stochastic partition between the base and collector terminals.

2.8 Noise suppression bandwidth

The spectral dependence of the suppressed recombination noise in macroscopic and microscopic junctions has been extensively measured [34–36]. The noise-suppression bandwidth was found [34] to be the same as that for external current modulation as in Fig. 4 and as expected for the leaky reservoir model.

However, as first pointed out by Yamamoto *et al.* [35, 37] and as shown in Fig. 4, contrary to the predictions of the leaky reservoir model, the bandwidth of the suppressed noise varies greatly with the DC current. It approaches the recombination lifetime limited bandwidth $B = 1/2\pi\tau_r = 1/2\pi r_d\, C_{diff}$ only in the limit of high current.

The reservoir model does not account for this variation. It also does not explain the successful suppression of noise in the limit of low current injection, when the diffusion capacitance becomes much less than the junction depletion capacitance, and when the characteristic time scale associated with injection across the depletion layer becomes much longer than the recombination lifetime (the reservoir leakage time scale). To understand this it is necessary to consider the detailed charge injection and transport across the depletion layer as well as the generation–recombination and diffusion processes that take place in homojunctions and heterojunctions according to accepted junction theory [23]. Despite this conceptual difficulty with the simple reservoir diffusion model, the equivalent circuits of Fig. 3 can be retained and the bandwidth variation of Fig. 4. can be well reproduced by the simple expedient of identifying the capacitance C as the total junction capacitance, the sum of the depletion and diffusion capacitances [24, 27] so that the noise suppression bandwidth becomes

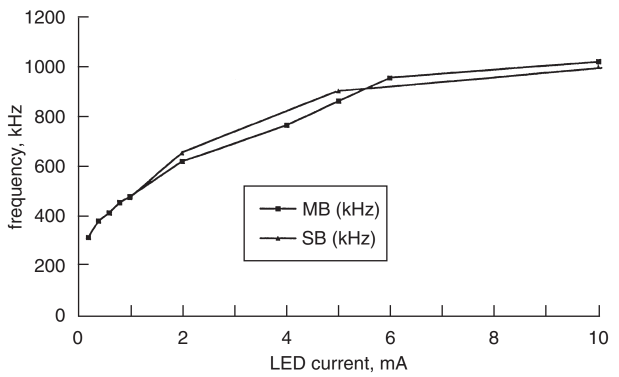

Fig. 4 *Noise suppression bandwidth and the current modulation bandwidth of a cooled commercial light-emitting diode (Hamamatsu L2656) showing the dependence on DC bias current supplied from a high impedance source (Zhang et al. [34])*
MB = modulation bandwidth; SB = squeezing bandwidth

$$\begin{aligned} B &= 1/2\pi r(C_d + C_{dep}) \\ &= 1/2\pi(kTC_{dep}/Ie + \tau) \end{aligned} \qquad (12)$$

In the limit of low injection the recombination dynamics thus appear to be irrelevant and some other noise suppression process must operate. This was identified by Imamoglu *et al.* [37] as the space-charge-regulated injection of carriers across the junction and the consequent suppression of pump noise by a 'macroscopic Coulomb blockade' process, named by analogy with the single-electron Coulomb blockade phenomenon found in mesoscopic junctions.

3 Physical processes

3.1 Historical development

Early theoretical treatments of terminal current shot noise generated in semiconductor junctions in the mid 1950s [1–5] initially attributed its origin to the random transport of charge carriers across the depletion layer. This was a natural conception because of the apparent similarity of the situation in the depletion layer to that in a temperature limited vacuum diode to which Schottky's analysis of 1918 presumably applied [38].

The physical basis of this analogy was challenged by Buckingham, Faulkner, Robinson and others in 1974 [6–8]. They attributed the shot noise to two other mechanisms operating in the neutral (bulk) regions of the structure; thermal fluctuations in minority charge carrier diffusion and fluctuations in the rates of generation and recombination. Their model, unlike earlier models, is consistent with the Shockley diffusion model [23] of charge carrier transport in a forward-biased homojunction. Buckingham and Faulkner developed the diffusion theory to provide a physical origin for the full shot noise observed in the terminal current of a long homojunction diode operated with a fixed terminal voltage. Their analysis demonstrated that homojunction terminal current shot noise did not originate in the depletion layer. They examined three sources of terminal current noise in detail: (*a*) generation–recombination noise; (*b*) thermal diffusion noise; and (*c*) depletion layer noise. The Buckingham diffusion noise model replaced the 'corpuscular' model with a physical model consistent with the Shockley theory of an ideal junction. It was later extended by Yamamoto *et al.* [17, 20, 25] and refined by Kobayashi *et al.* [27] for n^+-p single heterojunction diodes to successfully reproduce the dependence of noise suppression bandwidth on injection current.

3.2 Thermal diffusion noise and g–r (generation–recombination) noise

According to the Buckingham diffusion model as extended by Kim, Yamamoto *et al.*, thermal diffusion and generation–recombination events lead to fluctuations in the spatial distribution of minority charge carriers. These drive relaxation currents across the junction and also within the bulk material that act to restore the unperturbed steady-state distribution. Similarly, in the event of the generation or recombination of an electron–hole pair, there will be a perturbation of the minority carrier distribution and minority currents will flow to restore equilibrium. The total terminal current noise that flows in the external circuit is the sum of these independent diffusion and g–r noise currents.

For equilibrium electron density n_{po} at the p-type edge of the depletion layer, Einstein diffusion constant D_n, junction area A , electron mean free path l_f, diffusion length L_n and junction voltage V_j the forward DC current is

$$I = (eAD_n/L_n)(n_p - n_{po}) = Ne/\tau_r \tag{13}$$

with excess minority carrier population $N=(n_p-n_{po})L_n$ and reverse saturation current $I_s=(eAD_n/L_n)n_{po}$.

The DC junction current is due entirely to recombination of the excess minority charge in the p$^+$-type region, and the full short-circuit terminal shot noise spectral density level is reached when the terminal voltage, and therefore the junction voltage, is initially fixed by the external bias source. In this case the electron density on the p-side of the junction and at the p-side terminal are also pinned by the bias voltage. Subject to these boundary conditions, the current noise can then be calculated from the fluctuations in the electron distribution on the p-side for both heterojunction and homojunctions. The spectral density

$$S_i(\Omega) = 2e^2 L_n.(n_p + n_{po})/\tau_r = 2e(I + 2I_s) \tag{14}$$

is then entirely accounted for by the Poissonian recombination of a fixed population of $(N+2N_0)\approx N$ excess electrons with fixed lifetime τ_r, consistent with the heuristic model already discussed and illustrated in Figs. 1 and 2. The correlation function for the terminal current noise is given by

$$\langle F_i(t)F_{it}(t')\rangle = 2(Ie).\delta(t - t')/(1 + R_s/r_d)^2 \tag{15}$$

which shows the full shot noise current for $R_s \ll r_d$, as expected. The net result of this analysis is that all the external current noise can be accounted for in terms of diffusion and g–r noise in the bulk recombination region, where recombination provides the randomly fluctuating sink for all the minority carriers that have crossed the depletion layer. Why then is there no significant noise contribution associated with charge crossing the depletion layer of a macroscopic homojunction or heterojunction operating under normal bias conditions?

3.3 Depletion layer noise

According to the Shockley theory [23], extremely large forward (I_f) and backward (I_b) diffusion currents actually flow across the depletion layer. These are both much greater than the external terminal current, by a factor equal to the ratio of the minority carrier diffusion length to the mean free path, typically by several orders of magnitude [7, 8, 20].

The terminal current $I=(I_f-I_b)$ is the difference between these two large currents. It is relatively so small that it can be neglected in conventional calculations of the charge carrier distribution across the junction that assume that the quasi-Fermi levels are constant across the depletion layer and separated by the applied terminal voltage. The flux of majority carriers (electrons from the n-side of an abrupt n$^+$-p homojunction, for example) which constitutes the forward diffusion current I_f consists of those electrons in the bulk n$^+$ material incident on the junction which have sufficient energy to surmount the junction potential barrier. The flux of minority carriers, which constitutes the backward current, comprises all those electrons incident on the junction from the p-side. The existence of these large counterflowing charge carrier fluxes was noted by Chenette in his 1967 review [4], who recognised that they had no external effect on the low-frequency noise and by Robinson in his 1974 monograph [8], who emphasised their physical significance. Buckingham [7] introduced a differential depletion layer resistance

$$r_j = kT/eI_f \tag{16}$$

to account for the resulting small voltage drop $\Delta V \ll kT/e$ across the depletion layer. For strongly biased homojunctions this resistance is negligible in comparison with the diffusion resistance $r_d=kT/eI$. However it provides the relaxation mechanism whereby equilibrium is quickly restored following the passage of a carrier across the junction.

The passage of an electron across the junction raises the electron concentration at the p-side edge of the junction and this departure from the steady-state results in two relaxation current flows; one back across the junction, causing an increase in I_b, the other through the p-region, causing an increase in forward current I_f. The source of the current noise due to the $(I_f+I_b)/e$ junction-crossings per second, each generating transient current $\pm e\delta(t)$, is the depletion conductance $eI_f/kT \approx eI_b/kT$. This is so much larger than the diffusion conductance that its external shot noise current contribution, driven by voltage noise density $4kTr_j$, is evidently completely negligible in comparison with the total shot noise voltage $2kTr_d$ due to thermal diffusion and generation–recombination noise in the bulk material.

With typical values of L_n and l_f, the forward and back diffusion currents will each be of the order of amps for a silicon or germanium homojunction carrying a terminal current of a few milliamps. The corresponding diffusion resistance r_d in Fig. 5 will be several ohms while the forward and back resistances $r_f=r_b$ will only be of the order of 10^{-2} ohms. The independent mean square shot noise voltages $\langle v_f^2\rangle = \langle v_b^2\rangle = \langle i_f^2 r_f^2\rangle$ will then both be a factor l_f/L_n (at least two orders of magnitude) less than the recombination noise voltage $\langle v_{sn}^2\rangle$ and will therefore contribute very little to the terminal shot noise current, so that the corpuscular concept, that diode shot noise 'originates' in random crossings of the depletion layer, is not valid in the macroscopic diffusion limit.

When the external bias resistance in Fig. 5 is set to zero, it is evident that at low frequencies, neglecting the ohmic device resistance, the shot noise voltage generator $v_{sn}(t)$ will drive the terminal current to the full shot noise level $\langle i_{sn}^2\rangle = \langle v_{sn}^2\rangle / r_d^2$.

Although the net depletion layer current (the injection current) evidently does carry full shot noise, it originates in the bulk regions of the junction. The Buckingham model accounts for the terminal current noise for a voltage–biased diode in terms of thermal diffusion noise and generation–recombination noise components. Together these constitute

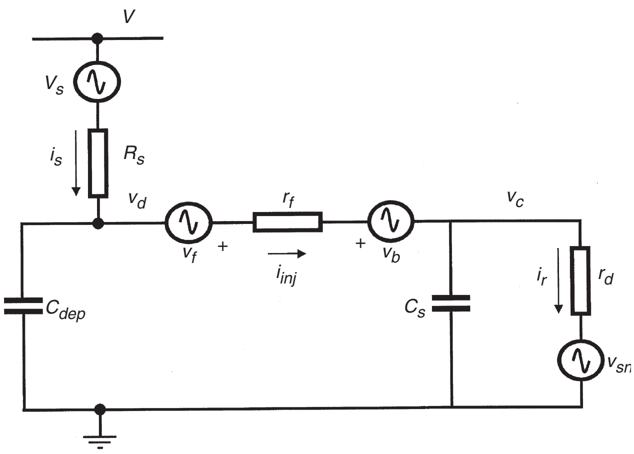

Fig. 5 *Noise equivalent circuit of a moderately forward biased junction diode showing three shot noise voltage sources associated with: forward diffusion (v_f) and back diffusion (v_b) across the depletion layer and recombination (bulk diffusion plus g–r) noise voltage (v_{sn}) in the 'diffusion limit' [25] (Edwards, [19])*

the shot-noise-limited 'pump' current noise which flows when the diode terminals are short-circuited by a low impedance bias circuit. Nevertheless, the 'bottom line' random (Poissonian) fluctuation noise in the spontaneous electron–hole recombination rate ensures that the low-frequency terminal current contains full shot noise. Spontaneous recombination therefore has prior claim as the 'origin' of shot noise in a strongly forward-biased homojunction.

3.4 Macroscopic coulomb blockade

The external current noise must also be suppressed if the electron–hole recombination rate fluctuations are to be suppressed, because any current variation will modulate the population number and hence the recombination rate. This must be so because the low-frequency terminal current $I(t) = Q(t)/\tau = eN(t)/\tau$ in the long diode model. From a circuit level viewpoint, the shot noise limited pump current modulates the light output of a light-emitting diode operated at constant voltage and generates the full recombination shot noise in the emitted photon stream. Suppression of the external terminal current noise is therefore a *necessary* but not a *sufficient* condition for recombination noise suppression. For example, the terminal voltage will be pinned at high frequencies leading to full recombination shot noise (observed photoelectrically) although the external current noise may still be strongly suppressed.

Initial discussions of recombination noise suppression in light emitting diodes and laser diodes [15–17] assumed that the use of a high impedance, 'constant current' bias source would automatically suppress the pump noise associated with the injection of charge carriers into the active recombination region of the diode. In the case of moderately strongly pumped homojunctions, the forward/back diffusion mechanism acts to suppress depletion layer transport noise independently of the bias impedance. However, Imamoglu and Yamamoto [37] pointed out that for weak injection, forward emission dominates and the Buckingham/Faulkner mechanism no longer operates. The injection of carriers across the junction by diffusion or thermionic emission into the active region generates shot noise and its suppression in the high impedance regime must be then explicable in terms of the dynamics of transport across the depletion layer. They identified the importance of pump noise suppression in the generation of sub-Poissonian photon fluxes and asked the question: how is the stochastic injection noise suppressed? They concluded that injection current-induced voltage fluctuations at the junction provide a negative feedback mechanism to smooth out the carrier injection rate, as in the case of space-charge-limited thermionic electron current noise in vacuum diodes [24]. They pointed out that the macroscopic junction voltage could only respond significantly to large charge fluctuations, not to individual injection events. This recognition of a macroscopic or collective Coulomb blockade effect bridged the conceptual gap between single-electron Coulomb blockade suppression of thermionic emission noise in mesoscopic junctions and noise suppression in macroscopic junctions.

The macroscopic junction voltage drop due to the injection of N_i carriers across a macroscopic junction is $\Delta V_j = N_i e/C_{dep}$. As a result the forward injection current will on average initially decrease to a fraction $\exp[-e\Delta V_j/kT] = \exp[-N_i\beta]$ of its initial value where $\beta = e^2/kTC_{dep}$. Individual carrier injection events will therefore not affect subsequent injections because the ratio of the depletion layer charging energy to the thermal energy $\beta = e^2/kT \ll 1$ in a macroscopic junction. The injection process might then be expected to be a random (Poisson) point process despite the fact that the current noise in the external circuit is suppressed to thermal levels by a high bias resistance. However, collective effects will evidently be significant when $N_i\beta \approx 1$, that is when the number of injected carriers N_i is of the order of $1/\beta = kTC_{dep}/e^2$. Since the mean injection rate is I/e, this establishes a characteristic time scale τ_i such that $\tau_i I/e = N_i = 1/\beta$, on which the junction voltage responds and provides negative feedback to regulate the injection rate where

$$\tau_i = e/\beta I = \tau/\beta = kTC_{dep}/Ie = r_d\, C_{dep} \qquad (17)$$

The charging energy $N_i e^2/2C_{dep}$ at the junction associated with the injection of N_i electrons into the active layer drops the junction potential and raises the Coulomb barrier against subsequent charge injection. This occurs if $N_i e^2/2C_{dep} > kT$. It results in a stream of antibunched electrons on a characteristic time scale $\tau_i = rC_{dep} = kTC_{dep}/eI$.

As the injection current is lowered, this time scale lengthens and exceeds the recombination time so that there is negligible charge storage and no associated spontaneous noise. Recombination is relatively rapid, electrons recombine shortly after injection and the recombination statistics are determined by the injection statistics. The corresponding bandwidth over which noise suppression occurs is then $B = eI/2\pi kT\, C_{dep}$. For low injection this will be less than the measured recombination bandwidth as in Fig. 4. In this situation the shot noise originates in the forward transport of electrons across the depletion layer as in van der Ziel's original models. With high impedance bias it will be suppressed by the space-charge-induced junction voltage fluctuations.

3.5 Junction voltage dynamics

In their detailed physical analysis of noise in p^+n heterojunction light emitters, Kim and Yamamoto [25] decompose the diode pump current noise into two components: a Markovian carrier injection current and a band-limited current fluctuation regulated by charging effects at the junction. It is this latter current which partially cancels the low-frequency components of the shot noise arising from the Markovian carrier injection and subsequent recombination. The junction current is carried by electrons injected from the n-layer across the depletion layer into an active p-type layer with subsequent radiative recombination. As in the ideal Shockley model [23] no carrier recombination is assumed to take place within the depletion region and the Sah–Noyce Shockley noise current is ignored.

The forward injection of carriers across the junction is modelled as the thermal diffusion of electrons from the n-type layer to the active p-type region, although the authors point out that a thermionic emission transport model gives similar results. A large flux of electrons will also diffuse back in the opposite direction and the net diffusion current is given by the difference between these forward and backward injection currents as in the homojunction model discussed by Buckingham and Faulkner.

3.6 Injection noise

This current can be written to explicitly contain two stochastic Langevin terms representing forward and backward charge carrier injection noise. The noise associated with the injection process will therefore depend on the dynamics of these two currents, both of which are usually very much higher than the net diffusion current in a strongly biased homojunction. The external current $I(t)$

therefore consists of the charging current $C_{dep}\ dV_j/dt$ plus the net charge transport current, $I_f(t) - I_b(t)$. For a standard macroscopic junction LED operating at temperatures above a few kelvins, the factor $\beta = e^2/kT\ C_{dep}$ will be much less than unity and single injection events will not have any significant effect on the forward current. In this limit the forward diffusion current will then comprise a stochastic injection current plus a slowly varying current averaged over a large number of injection events N_i, typically of the order of 10^8, which varies with the time-dependent junction voltage.

3.7 Hiroshima (backward pump) model

The publication of the Stanford analysis by Kim and Yamamoto [25] stimulated further theoretical and experimental work on sub-Poissonian light emission from macroscopic heterojunctions. Kobayashi *et al.* [27] working at Hiroshima University noticed that their measurements of the noise suppression bandwidth obtained with commercially available heterojunction LEDs were not well fitted by the Stanford model. The Stanford model appears to be a good fit in the low and high current ranges, but not the intermediate current ranges. The Hiroshima group examined the transition from thermionic emission at low pump currents to the macroscopic diffusion limit at high currents. They were able to account for their experimental results over a wide range of pump currents with a unified model of injection and recombination based on a detailed re-examination of the backward pump process.

The LED used in the Hiroshima work behaved essentially as a single p^+-N junction device; the active region layer and the adjacent P-type layer were much more highly doped than in the N-type region, so that most of the junction voltage drop fell across the p-N junction. As in the Stanford analysis, the device was therefore characterised by electron transport from the N-type region to the p-type active layer and by the recombination dynamics within that active region. Unlike the situation envisaged in the Stanford model which was based on the classical Shockley/Buckingham diffusion model devised for homojunctions, Kobayashi *et al.* [27] suggest that the injected electrons may not completely thermalise to the conduction band edge because of hot carrier effects and band-tail filling. As a result, electrons with energies greater than the barrier height may have higher probability of returning to the N-side. In their measurements the injected electron density was estimated to be much lower than the hole concentration, permitting a linear recombination model.

Kobayashi *et al.* defined a parameter $\alpha_o = I_{bo}/I_{fo}$ and a differential ratio $\alpha_d = dI_{bo}/dI_{fo}$ to characterise the ratio of the backward (BP) and forward (FP) injection currents. The spectral Fano factor for the recombination noise, found by solving the Langevin equations, was then found as a function of α_o, α_d, and $\tau_i = C_{dep}\ r_d$, the junction response time.

The case $\alpha_o = \alpha_d = 1$ recovers the macroscopic diffusion limit previously discussed in which the pump and recombination processes are strongly coupled and for which $f_{3dB} = 1/2\pi\ (\tau_r + \tau_i)$ (see (12). For the low injection case $\alpha_o = \alpha_d = 0$, the back current is zero and the forward injection and recombination processes are viewed as separate cascaded processes as in the thermionic emission limit treated by Imamoglu and Yamamoto [37]. This comparison led the Hiroshima group to suggest a model in which the backward pump rate increased monotonically but in a nonlinear fashion from zero at low currents to equality with the forward injection rate at high currents. They obtained good agreement with the measured bandwidth dependence on current between $\alpha \approx 0$ (the thermionic limit) and $\alpha \approx 1$ (the diffusion limit).

4 Conclusions

Photoelectric measurement and analysis of the fluctuations in the recombination light from semiconductor junction light emitters have provided new insights into the internal shot noise generating processes in semiconductor junction devices. In particular, the recombination shot noise generation and suppression mechanisms that operate in macroscopic junctions have been identified and are now better understood.

These measurements show that shot noise originates in the bulk recombination regions of strongly forward biased ideal bipolar junctions, as proposed by Buckingham and Faulkner. The shot noise generated in strong charge injection across macroscopic depletion layers does not significantly contribute to either the external circuit recombination noise current or to the optical recombination noise itself, contrary to early corpuscular shot noise models. The smoothing mechanism for this suppression is provided by the buffer of minority charge stored in the vicinity of the junction which results in extremely large forward and back diffusion currents across the junction. In this 'diffusion limit' these heavy currents raise the conductance of the depletion layer and effectively short-circuit its shot noise current. However, as the forward injection current is reduced, the stored charge and back diffusion current both decrease. The transport of charge across the junction then becomes predominantly forward and in the 'thermionic limit' all the shot noise originates in depletion layer transport as originally proposed by van der Ziel and Becking [2].

Yamamoto and coworkers showed that the recombination shot noise in the external circuit current and in the emitted photon flux can both be suppressed by driving a bipolar junction from a high impedance source. This allows the junction voltage to fluctuate freely. In the 'diffusion limit', this allows the charge population of the 'leaky reservoir' to fluctuate on time scales longer than the recombination time. This rate compensation mechanism suppresses the low-frequency spectral density of the recombination shot noise. In the 'thermionic limit', the reservoir is virtually empty and charge carriers effectively recombine immediately following injection. In this limit, realised for mesoscopic junctions and for weak injection across macroscopic junctions, the fluctuating junction potential modulates the forward injection rate directly and results in low-frequency shot noise suppression. This latter mechanism is the macroscopic Coulomb blockade effect. These processes have all been shown to suppress electronic and photonic noise in the laboratory with potential real world applications in cryptography, communications, computing and metrology.

Finally, measurement and analysis of the collector current shot noise in optocouplers and photon transport transistors in which photons, rather than charge carriers, are collected gives results that are identical with those measured for bipolar junction transistors. This agreement leads naturally to a general corpuscular shot noise model incorporating the processes of high impedance shot noise suppression and partition noise insertion that applies equally well to both photonic and electronic devices, again breathing new life into the early corpuscular noise models of van der Ziel and Becking [2].

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6 References

1 Van Der Ziel, A.: 'Theory of shot noise in junction diodes and junction transistors', *Proc IRE*, 1955, **43**, pp. 1639–1646
2 Van Der Ziel, A., and Th. Becking, A. G.: 'Theory of junction diode and junction transistor noise', *Proc. IRE*, 1958, **46**, pp. 589–594
3 Van Der Ziel, A.: 'Fluctuation phenomena in semiconductors' (Butterworth, London, UK, 1959)
4 Chenette, E.R.: 'Noise in semiconductor devices', *Adv. Electron. Electron Phy.*, 1978, **23**, pp. 303–345; (Adv. Electron. Electron Phys., 1967, p. 306)
5 Giacoletto, L.J.: in 'Transistors 1' (RCA Labs., Princeton, NJ, USA, 1956), p. 296
6 Buckingham, M.J., and Faulkner, E.A.: 'The theory of inherent noise in p-n junction diodes and bipolar transistors', *Radio Electron. Eng.*, 1974, **44**, pp. 125–140
7 Buckingham, M.J.: 'Noise in electronic devices and systems' (Wiley, New York, 1983)
8 Robinson, F.N.H.: 'Noise and fluctuations in electronic devices and circuits' (Clarendon Press, Oxford, 1974), p. 99 (and 237–239)
9 Edwards, P.J.: 'Semiconductor junction noise revisited: where have all the physical noise sources gone?', *Fluct. Noise Lett.*, 2001, **1**, (3), pp. C15–C19
10 Edwards, P.J.: 'Brief history of recombination noise in semiconductor junction devices', *Proc. SPIE - Int. Soc. Opt. Eng.*, 2003, **5113**, pp. 204–216
11 Tapster, P.R., Rarity, J.G., and Satchell, J.: 'Generation of sub-Poissonian light by high-efficiency light-emitting diodes', *Europhys. Lett.*, 1987, **4**, pp. 293–299
12 Edwards, P.J.: Proc. Fourth Australian Conf. on II-VI semiconductor compounds, 1990, Abstract W9, p. 57
13 Edwards, P.J., and Pollard, G.H.: 'Quantum noise-correlated operation of electrically coupled semiconductor light-emitters', *Phys. Rev. Lett.*, 1993, **69**, pp. 1757–1760
14 Edwards, P.J.: 'Sub-Poisson light from GaAlAs infrared emitting diodes', *Int. J. Optoelectron.*, 1991, **6**, (1/2), pp. 23–28
15 Machida, S., Yamamoto, Y., and Itaya, Y.: 'Observation of amplitude squeezing in a constant-current-driven semiconductor laser', *Phys. Rev. Lett.*, 1987, **58**, pp. 1000–1003
16 Yamamoto, Y., Machida, S., and Nillson, O.: 'Amplitude squeezing in a pump-noise-suppressed laser oscillator', *Phys. Rev. A*, 1986, **34**, (5), pp. 4025–4042
17 Yamamoto, Y., and Machida, S.: 'High impedance suppression of pump fluctuations and amplitude squeezing in semiconductor lasers', *Phys. Rev. A*, 1987, **35**, pp. 5114–5130
18 Edwards, P.J.: 'Reduction of optical shot noise from light-emitting diodes', *IEEE J. Quantum. Electron.*, 1993, **29**, pp. 2302–2304
19 Edwards, P.J.: 'Sub-poissonian recombination noise in macroscopic and mesoscopic semiconductor junctions', in Balandin, A. (Ed.): 'Noise and fluctuation control in electronic devices' (American Scientific Publishers, 2002), Chap. 16
20 Kim, J., Somani, S., and Yamamoto, Y.: 'Nonclassical light from semiconductor lasers and LEDs' (Springer, 2001)
21 Kikuchi, K., and Kakui, M.: 'Reduction of shot noise with light emitting diodes', *IEEE J. Quantum. Electron.*, 1992, **28**, (7), pp. 1626–1630
22 Teich, M.C., and Saleh, B.E.A.: 'Photon bunching and anti-bunching', *Prog. Opt.*, 1988, **26**, pp. 1–104
23 Shockley, W.: 'The theory of p-n junctions in semiconductors and p-n junction transistors', *Bell Syst. Tech. J.*, 1949, **28**, pp. 453–489
24 Edwards, P.J.: 'Sub-Poissonian electronic and photonic noise generation', *Aust. J. Phys.*, 2000, **53**, pp. 179–192
25 Kim, Y., and Yamamoto, Y.: 'Theory of noise in p-n junction light emitters', *Phys. Rev. B, Condens. Matter*, 1997, **55**, pp. 9949–9959
26 Shinozaki, G., Hirano, T., Kuga, T., and Yamanashi, M.: '3 dB wideband squeezing in photon number fluctuations from a light emitting diode', *Jpn. J. Appl. Phys.*, 1997, **36**, pp. 6350–6352
27 Kobayashi, M., Yamanishi, M., Sumitomo, H., and Kadoya, Y.: 'Influence of the backward-pump process on photon–number squeezing in a constant-current-driven heterojunction LED: transition from thermionic emission to diffusion limits', *Phys. Rev. B, Condens. Matter*, 1999, **60**, pp. 16686–16700
28 Lathi, S., Tanaka, K., Morita, T., Inoue, S., Kan, H., and Yamamoto, Y.: 'Transverse junction stripe GaAs – AlGaAs lasers for squeezed light generation', *IEEE J. Quantum. Electron.*, 1999, **35**, (3), pp. 387–394
29 Edwards, P.J., Cheung, W.N., Uddin, A., Ganeshkumar, G., Tanaka, K., Morita, T., Kan, H., and Yamamoto, Y.: 'Photon transfer incorporating an amplitude-squeezed transverse junctin stripe laser'. Conf. Digest CLEO 2000, Nice, France, 2000, Vol. 38
30 Edwards, P.J., Cheung, W.N., Uddin, A., Ganeshkumar, G., Tanaka, K., Morita, T., Kan, H., and Yamamoto, Y.: 'Quantum noise-suppressed operation of TJS and SQW ridge-waveguide diodes in photon transistor configurations'. Proc. COMMAD 2000, Melbourne, Australia, Dec. 2000, (IEEE, 2002) pp. 101–104)
31 Rutz, R.F.: 'Transistor-like device using optical coupling between diffused p-n junctions in GaAs', *Proc. IEEE*, 1963, **51**, pp. 470–471
32 Cheung, W.N., and Edwards, P.J.: 'Characteristics of a class of new opto-coupler amplifiers with positive feedback', *IEEE J. Quantum Electron.*, 1996, **32**, pp. 502–512
33 Edwards, P.J., McDonald, R., and Cheung, W.N.: 'Semiconductor junction noise revisited'. Proc. 2nd Int. Conf. on Unsolved Problems of Noise (UPoN'99), Adelaide, Australia, 1999, (AIP, USA, 2000) pp. 415–421
34 Zhang, T., Edwards, P.J., Cheung, W.N., and Sun, H. B.: 'Electrical characterization of a quantum noise suppressed light emitting diode'. Tech. Digest of CLEO/ Pacific Rim Conf., 10–14 July, 1995, pp. 71–72
35 Kim, J., Kan, H., and Yamamoto, Y.: 'Macroscopic coulomb blockade effect in a constant-current-driven light-emitting diode', *Phys. Rev. B, Condens. Matter*, 1995, **52**, pp. 2008–2012
36 Abe, J., Kuga, T., Hirano, T., Kobayashi, M., and Yamanishi, M.: 'Wideband squeezing in photon number fluctuations from a high speed light-emitting diode', *Opt. Express*, 2000, **7**, pp. 215–221
37 Imamoglu, A., and Yamamoto, Y.: 'Noise suppression in semiconductor p-i-n junctions: transition from macroscopic squeezing to mesoscopic coulomb blockade of electron emission processes', *Phys. Rev. Lett.*, 1993, **70**, pp. 3327–3330
38 Schottky, W.: 'Spontaneous current fluctuations in various conductors', *Ann. Phys. (Germany)*, 1918, **57**, pp. 541–552