Published for SISSA by Springer

Received: *October 6, 2010*
Accepted: *November 2, 2010*
Published: *November 18, 2010*

On the tensionless limit of string theory, off-shell higher spin interaction vertices and BCFW recursion relations

Angelos Fotopoulos[a] **and Mirian Tsulaia**[b,c,1]

[a] *Dipartimento di Fisica Teorica dell'Università di Torino and INFN Sezione di Torino, via P.Giuria 1, I-10125 Torino, Italy*

[b] *Department of Mathematical Sciences, University of Liverpool, Liverpool, L69 7ZL, United Kingdom*

[c] *Centre for Theoretical Chemistry and Physics, Massey University of Auckland, Private Bag 102904, 0745 Auckland, New Zealand*

E-mail: foto@to.infn.it, tsulaia@liv.ac.uk

Abstract: We construct an off-shell extension of cubic interaction vertices between massless bosonic Higher Spin fields on a flat background which can be obtained from perturbative bosonic string theory. We demonstrate how to construct higher quartic interaction vertices using a simple particular example. We examine whether BCFW recursion relations for interacting Higher Spin theories are applicable. We argue that for several interesting examples such relations should exist, but consistency of the theories might require that we supplement Higher Spin field theories with extended and possibly non-local objects.

Keywords: Gauge Symmetry, BRST Symmetry, String Field Theory

ArXiv ePrint: 1009.0727

[1] Associate member of the Centre of Particle Physics and Cosmology, Ilia State University, 0162, Tbilisi, Georgia.

doi:10.1007/JHEP11(2010)086

JHEP11(2010)086

Contents

1 Introduction

Higher Spin (HS) gauge theories (see [1–9] for recent reviews) are usually formulated in two different ways: in frame-like [10–20] or in metric-like [21–77] approaches. Although a formulation of free dynamics of various representations of Poincare and anti de Sitter groups in both approaches is quite a nontrivial task, the most challenging problem is the construction and study of interactions between both massless and massive Higher Spin fields on flat and curved backgrounds.

A landmark has been reached in [10–14] with the understanding that, the anti de Sitter background can accommodate consistent self interactions of massless Higher Spin fields. The interaction involves an infinite tower of massless Higher Spin fields and is non local. Moreover the presence of an anti de Sitter background, where no S-matrix can be defined, allows to naturally bypass Coleman-Mandula no-go theorem. These results, apart form being remarkable in their own right, can also be extremely useful for a better understanding of String/M theory in particular in the context of AdS/CFT correspondence [78–85].

The interaction of massless Higher Spin fields on a flat background has always been considered to be more problematic than the interaction on AdS space. However, this problem has recently been extensively discussed in [42–58] and several interesting cubic vertices have been obtained. These results indicate that one can possibly have consistent interacting theories of massless Higher Spin fields on a Minkowski background as well, provided that, as in the case of an AdS background, one has an infinite number of fields and the interactions are non local.

It is extremely important to find a guiding principle to construct interactions on flat space time. A possible approach is to consider perturbative String Theory as a "laboratory" for studying interactions between massless Higher Spin fields [48–55, 57, 58]. Obviously since in the usual tensile string theory all Higher Spin fields are massive, one needs to go to a limit where the masses of Higher Spin fields go to zero. However the high energy limit of string theory [86–88] is still to be better understood.[1]

In a recent paper [55], the authors constructed cubic vertices for HS fields via the high energy limit of string perturbation theory. Their result has as a part of it the vertex constructed in [48] via the high energy limit of Open String Field Theory (OSFT), but includes additional terms which grant the new vertex with a non-abelian structure. These results are extremely important to further study the problem of interactions between massless fields on a flat background, since the well developed technique of string perturbation theory can allow the study of higher order interactions and perform nontrivial consistency checks of the theory. One of the aims of the present paper is to demonstrate how one can construct vertices which describe a consistent cubic interaction of massless bosonic Higher Spin fields, serving as an off-shell completion of the on-shell vertex derived in [55]. We are using the BRST method for interacting higher spin fields, which can be formulated in flat and in AdS spaces [27, 68–71]. This method is essentially based on the requirement of gauge invariance and the abelian gauge transformations of the free theory are deformed to nonlinear (generically nonabelian) transformations to obtain nontrivial cubic and higher order interactions.

Having formulated an off-shell description for a system which describes cubic interactions between massless Higher Spin fields belonging to reducible representations of the Poincare group, one can further consider higher order interactions for massless Higher Spin Fields following standard perturbation theory. This consideration will hopefully improve our understanding of the problem of interactions in Higher Spin theories on Minkowski background, since this approach can be further generalized to higher (hopefully arbitrary) orders in the coupling constant. Therefore we consider the present article as a step towards a better understanding and towards further consistency checks of the interacting theory of massless Higher Spin fields on a flat background. As a particular example we show how to construct a quartic interaction vertex from a cubic Lagrangian described in [95]. This problem, although being technically simpler than the problem of construction of quartic interaction vertices from cubic ones of the type [55], is still quite instructive and contains some lessons on how to deal with more complicated systems.

In the second part of the paper we will follow a different approach to the problem based on the S-matrix rather than the Lagrangian itself. Recently, there has been remarkable progress in exploring the properties of the S-matrix for tree level scattering amplitudes. Motivated by Witten's twistor formulation of $\mathcal{N}=4$ Super-Yang-Mills (SYM) [96], several new methods have emerged which allow one to compute tree level scattering amplitudes

[1] Related work on the tensionless limit of string theory has been done in [89–94].

for gauge and gravity theories. In particular, the CSW method of [97] has demonstrated how one can use the Maximum Helicity Violating (MHV) amplitudes of [98] as field theory vertices to construct arbitrary gluon amplitudes.

Analyticity of gauge theory tree level amplitudes has led to the BCFW recursion relations [99–101]. Specifically, analytic continuation of external momenta in a scattering amplitude allows one, under certain assumptions, to determine the amplitude through its residues on the complex plane. Locality and unitarity require that the residue at the poles is a product of lower-point on shell amplitudes. Actually the CSW construction turns out to be a particular application of the BCFW method [102].

The recursion relations of [100] are at the heart of many of the aforementioned developments. Nevertheless, for these relations the asymptotic behavior of amplitudes under complex deformation of some external momenta is crucial. When the complex parameter, which parametrizes the deformation, is taken to infinity an amplitude should fall sufficiently fast so that a pole at infinity will be absent.[2] Although naive power counting of individual Feynman diagrams seems to lead to badly divergent amplitudes for large complex momenta, it is intricate cancellations among them which lead to a much softer behavior than expected. In particular, gauge invariance and supersymmetry in some cases are responsible for these cancellations.

It is natural to wonder whether these field theoretic methods can be applied and shed some light into the structure of string theory amplitudes. This is motivated, in particular, by the fact that the $\mathcal{N} = 4$ SYM theory that plays a central role in the developments we described above, appears as the low-energy limit of string theory in the presence of D3-branes. In order to even consider applying the aforementioned methods to string scattering amplitudes, one needs as a first step to study the large complex momentum behavior of these amplitudes. Since generally string amplitudes are known to have excellent large momentum behavior, one expects that recursion relations should be applicable here as well. Nevertheless, one should keep in mind that although the asymptotic amplitude behavior might be better than any local field theory, the actual recursion relations are quite more involved. They require knowledge of an infinite set of on-shell string amplitudes, at least the three point functions, between arbitrary Regge trajectory states of string theory.

The study of the asymptotic behavior of string amplitudes under complex momentum deformations was initiated in [105] and further on elaborated in [106, 107]. In these works it was established, using direct study of the amplitudes in parallel with Pomeron techniques, that both open and closed bosonic and supersymmetric string theories have good behavior asymptotically, therefore allowing one to use the BCFW method. So, in light of the apparent relation of massless Higher Spin theories with the high energy limit of string theory one might be able to use these powerful tools in order to study the properties of HS theories themselves. We will argue that indeed the arguments which suggest that the boundary contribution under BCFW deformation of string amplitudes vanishes, extend to the corresponding Higher Spin amplitudes. The failure though of the criterion of [108]

[2] There has been though some recent progress [103, 104] in generalizing the BCFW relations for theories with boundary contributions.

will indicate that the Higher Spin theory on a flat background might be missing crucial ingredients required for its consistency at the full interaction level. In two examples we will derive some indications that consistent HS theories cannot rely solely on massless HS point particles in their spectrum but might require extra degrees of freedom like extended objects and/or non-local states with Pomeron like dynamics.

The paper is organized as follows: in section 2 we collect some of the main concepts of the BRST constructions for interacting massless Higher Spin fields. In section 3 we show, using the results of [27], how to construct a cubic off-shell interaction vertex which corresponds to the on-shell vertex [55], derived from the string perturbation theory. We also derive a possible generalization of this vertex to a system which contains bosonic massless fields which belong to reducible representations of the Poincare group with mixed symmetry. In section 4 we derive a quartic interaction vertex for the cubic Lagrangian constructed in [95]. We comment that also in this case, like the exact cubic vertex of [48], the algebra is abelian and it leads to a trivial interaction beyond the cubic level. In the last two sections we will give a short review of the BCFW method for field theories and we will apply this method on two different cubic couplings which will allow us to make some interesting observations on the possible theories with interacting HS particles on the flat background. We conclude with a summary of our results and some comments.

2 Basic equations

Let us start with the BRST charge for the open bosonic string (see for example [9] for more details)

$$\mathcal{Q} = \sum_{k,l=-\infty}^{+\infty} \left(C_{-k} L_k - \frac{1}{2}(k-l) : C_{-k} C_{-l} B_{k+l} : \right) - C_0, \tag{2.1}$$

perform the rescaling of oscillator variables

$$c_k = \sqrt{2\alpha'} C_k, \qquad b_k = \frac{1}{\sqrt{2\alpha'}} B_k, \qquad c_0 = \alpha' C_0, \qquad b_0 = \frac{1}{\alpha'} B_0, \tag{2.2}$$
$$\alpha^\mu_k \to \sqrt{k}\alpha^\mu_k$$

and then take a formal limit $\alpha' \to \infty$. In this way one obtains a BRST charge

$$Q = c_0 l_0 + \tilde{Q} - b_0 \mathcal{M} \tag{2.3}$$

$$\tilde{Q} = \sum_{k=1}^{\infty} (c_k l_k^+ + c_k^+ l_k), \quad \mathcal{M} = \sum_{k=1}^{\infty} c_k^+ c_k, \quad l_0 = p^\mu p_\mu, \qquad l_k^+ = p^\mu \alpha^+_{k\mu} \tag{2.4}$$

which is nilpotent in any space-time dimension. The oscillator variables obey the usual (anti)commutator relations

$$[\alpha^k_\mu, \alpha^{l,+}_\nu] = \delta^{kl}\eta_{\mu\nu}, \quad \{c^{k,+}, b^l\} = \{c^k, b^{l,+}\} = \{c^k_0, b^l_0\} = \delta^{kl}\,, \tag{2.5}$$

and the vacuum in the Hilbert space is defined as

$$\alpha^\mu_k|0\rangle = 0, \quad c_k|0\rangle = 0 \quad k > 0, \qquad b_k|0\rangle \ = \ 0 \qquad k \geq 0. \tag{2.6}$$

JHEP11(2010)086

Let us note that one can take the value of k to be any fixed number without affecting the nilpotency of the BRST charge (2.3). Fixing the value $k = 1$ one obtains the description of totally symmetric massless higher spin fields, with spins $s, s-2, ..1/0$. The string functional (named "triplet" [76, 77]) in this simplest case has the form

$$|\Phi\rangle = |\phi_1\rangle + c_0|\phi_2\rangle = |\varphi\rangle + c^+ \, b^+ \, |d\rangle + c_0 \, b^+ \, |c\rangle \tag{2.7}$$

whereas for an arbitrary value of k one has the so called "generalized triplet"

$$|\Phi\rangle = \frac{c^+_{k_1} \dots c^+_{k_p} b^+_{l_1} \dots b^+_{l_p}}{(p!)^2} |D^{l_1,\dots l_p}_{k_1,\dots l_p}\rangle + \frac{c_0 c^+_{k_1} \dots c^+_{k_{p-1}} b^+_{l_1} \dots b^+_{l_p}}{(p-1)!p!} |C^{l_1,\dots l_p}_{k_1,\dots k_{p-1}}\rangle, \tag{2.8}$$

where the vectors $|D^{k_1,\dots k_p}_{l_1,\dots l_p}\rangle$ and $|C^{k_1,\dots k_p}_{l_1,\dots l_p}\rangle$ are expanded only in terms of oscillators $\alpha^{\mu +}_k$, and the first term in the ghost expansion of (2.8) with $p = 0$ corresponds to the state $|\varphi\rangle$ in (2.7). One can show that the whole spectrum of the open bosonic string decomposes into an infinite number of generalized triplets, each of them describing a finite number of fields with mixed symmetries [66].

In order to describe the cubic interactions one introduces three copies ($i = 1, 2, 3$) of the Hilbert space defined above, as in bosonic OSFT [109–111]. Then the Lagrangian has the form

$$L \;=\; \sum_{i=1}^{3} \int dc_0^i \langle \Phi_i | \, Q_i \, | \Phi_i \rangle \;\; + g \left(\int dc_0^1 dc_0^2 dc_0^3 \langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 | | V_3 \rangle + h.c \right) \, , \tag{2.9}$$

where $|V_3\rangle$ is the cubic vertex and g is a coupling constant. The Lagrangian (2.9) is invariant at order g with respect to the nonabelian gauge transformations

$$\delta |\Phi_i\rangle = Q_i |\Lambda_i\rangle - g \int dc_0^{i+1} dc_0^{i+2} [(\langle \Phi_{i+1} | \langle \Lambda_{i+2} | + \langle \Phi_{i+2} | \langle \Lambda_{i+1} |) | V_3 \rangle] \, , \tag{2.10}$$

provided that the vertex $|V\rangle$ satisfies the BRST invariance condition

$$\sum_i Q_i |V_3\rangle = 0 \, . \tag{2.11}$$

The gauge parameter $|\Lambda\rangle$ in each individual Hilbert space has the ghost structure

$$|\Lambda\rangle = b^+ |\lambda\rangle \tag{2.12}$$

for the totally symmetric case, while the gauge parameters for the generalized triplets take the form

$$|\Lambda\rangle = \frac{c^+_{k_1} \dots c^+_{k_p} b^+_{l_1} \dots b^+_{l_{p+1}}}{(p!)(p+1)!} |\Lambda^{l_1,\dots l_{p+1}}_{k_1,\dots k_p}\rangle + \frac{c_0 c^+_{k_1} \dots c^+_{k_{p-1}} b^+_{l_1} \dots c^+_{l_{p+1}}}{(p-1)!(p+1)!} |\hat{\Lambda}^{l_1,\dots l_{p+1}}_{k_1,\dots k_{p-1}}\rangle.$$

Let us make some comments about the BRST charge (2.3). We can actually justify the way it was obtained from the BRST charge of the open bosonic string since its cohomologies correctly describe equations of motion for massless bosonic fields belonging to mixed symmetry representations of the Poincare group (see e.g. [66]). So taking the point of view that, in the high energy limit the whole spectrum of the bosonic string collapses to zero mass, which is now infinitely degenerate, one can take the BRST charge (2.3) as the one which correctly describes this spectrum.

3 Cubic vertex for HS interactions from the high energy limit of String Theory

In this section we will demonstrate how one can construct an off-shell extension of the cubic vertex suggested in [55]. The key point is that the free spectrum of string theory is given by the free Lagrangian of triplets in (2.9). This suggests that we use the on-shell vertex of [55] as an ansatz in the BRST method of the previous section for constructing interacting Lagrangians for triplets. The cubic vertex of [55] is an extension of the unmodified cubic vertex of [48, 95].

Let us describe first the proposed high energy limit of string theory of [48]. The cubic vertex in this case takes the form

$$V^1 = \exp(\ Y_{ij}\alpha^{i+}_\mu p^j_\mu + Z_{ij}c^{i+}b^j_0\ )\,. \tag{3.1}$$

The BRST invariance condition of the vertex (3.1) imposes the following condition on the coefficients Y_{ij} and Z_{ij}

$$\begin{aligned} Z_{i,i+1} + Z_{i,i+2} &= 0 \\ Y_{i,i+1} &= Y_{ii} - Z_{ii} - 1/2(Z_{i,i+1} - Z_{i,i+2}) \\ Y_{i,i+2} &= Y_{ii} - Z_{ii} + 1/2(Z_{i,i+1} - Z_{i,i+2}). \end{aligned} \tag{3.2}$$

There are two options available at this point. One is to consider the modification of the cubic Lagrangian (2.9) and gauge transformations (2.10) which involve quartic and higher order vertices. We shall consider this option in the section 4. Another option is to make this solution exact in all orders of the coupling constant g. For this we have to modify the solution above by an extra oscillator-dependent factor i.e., consider a modified ansatz [48]

$$|V\rangle = V^1 \times V^{\rm mod} c^1_0 c^2_0 c^3_0 |0\rangle_{123} \tag{3.3}$$

with

$$V^{\rm mod} = \exp\left(S_{ij}c^{i+}b^{j+} + \frac{P_{ij}}{2}\alpha^{i+}_\mu \alpha^{j+}_\mu\right). \tag{3.4}$$

The BRST invariance condition imposes the constraint on the coefficients S_{ij} and P_{ij}

$$\begin{aligned} S_{ij} = P_{ij} &= 0 \qquad i \neq j \\ P_{ii} - S_{ii} &= 0 \qquad i = 1,2,3 \end{aligned} \tag{3.5}$$

whereas the requirement that the solution (3.4) is exact in all orders in the coupling constant further restricts the value of the parameter S_{ii} to be equal to 1.

This vertex leads into abelian, but nonlinear gauge transformations. Therefore, one can verify using the technology developed in [71], that the four point function of such a theory turns out to be vanishing and makes the theory not a good framework for the study of non-trivial interacting HS theories at quartic and higher levels. In the next section we will try to relax the condition of an exact cubic vertex and allow a quartic vertex in our Lagrangian and possibly higher ones in an attempt to derive a non-trivial algebra and therefore non-trivial interactions.

JHEP11(2010)086

In an alternative method given in [55] one starts from the on-shell string amplitudes for the leading Regge trajectory of string theory and employs the tensionless limit through the study of decoupling of null states. An on-shell vertex obtained using this method has a leading term which is given by going on-shell in (3.1). The derived subleading terms involve an exponential of higher terms in the oscillators and lead in a non-trivial algebra of gauge transformations. In the following subsection 3.1 we will derive an off-shell expression of the vertex given in [55] for the leading Regge trajectory and in subsection 3.2 we will show how one can derive the cubic vertex for mixed symmetry HS fields. This result is an ansatz for the on-shell vertex for subleading Regge trajectories in the spirit of [55].

3.1 A cubic vertex for totally symmetric fields

Since the "leading" vertex (3.1) is BRST invariant on its own right we will consider the BRST invariance of the subleading modification suggested in [55]. We begin first with the simpler case of a vertex for totally symmetric fields. This means that we consider only one set of oscillators as in (2.5). Let us note that one can make the functionals $|\Phi_i\rangle$ matrix valued i.e., add Chan-Paton factors, but we shall not do so.

In order to make a connection with the on-shell vertex of [55] let us make the following ansatz

$$|V_3\rangle = e^v c_0^1 c_0^2 c_0^3 |0\rangle_{123} \,. \tag{3.6}$$

where, (we follow the conventions of [68]),

$$\begin{aligned} v = \; & X^{(1)}_{rstu}(\alpha^{r+}_\mu \alpha^{s+}_\mu)(\alpha^{t+}_\nu p^u_\nu) + X^{(2)}_{rstu}(c^{r+} b^{s+})(\alpha^{t+}_\mu p^u_\mu) + \\ & X^{(3)}_{rstu}(\alpha^{r+}_\mu \alpha^{s+}_\mu)(c^{t+} b^u_0) + X^{(5)}_{rstu}(c^{r+} b^{s+})(c^{t+} b^u_0) \end{aligned} \tag{3.7}$$

The terms in the expression above proportional to $X^{(1)}_{rstu}$ correspond to the operator $\mathcal{G}$ in [55]. The remaining terms are those required for extending off-shell their cubic on-shell description.

The coefficients $X^{(1)}_{rstu}, X^{(2)}_{rstu}, X^{(3)}_{rstu}$ and $X^{(5)}_{rstu}$ obey symmetry relations

$$X^{(1)}_{rstu} = X^{(1)}_{srtu}, \quad X^{(3)}_{rstu} = X^{(3)}_{srtu}, \quad X^{(5)}_{rstu} = -X^{(5)}_{tsru} \,. \tag{3.8}$$

They are constants to be determined form the BRST invariance condition of the vertex. The BRST invariance condition of the vertex implies

$$(2X^{(1)}_{rstu} p^r_\mu p^u_\nu - X^{(2)}_{rstu} p^s_\mu p^u_\nu) c^{r+} \alpha^{s+}_\mu \alpha^{t+}_\nu = 0, \tag{3.9}$$

$$(-X^{(3)}_{rstu} p^u_\mu p^u_\mu + X^{(1)}_{rstu} p^t_\mu p^u_\mu) c^{t+} \alpha^{r+}_\nu \alpha^{s+}_\nu = 0, \tag{3.10}$$

$$(-X^{(5)}_{rstu} p^u_\mu p^u_\mu + X^{(2)}_{rstu} p^t_\mu p^u_\mu) c^{r+} b^{s+} c^{t+} = 0, \tag{3.11}$$

$$(-X^{(2)}_{rtsu} b^t_0 p^u_\mu + 2X^{(3)}_{rstu} b^u_0 p^r_\mu - X^{(5)}_{rstu} b^u_0 p^s_\mu) c^{r+} c^{t+} \alpha^{s+}_\mu = 0, \tag{3.12}$$

$$X^{(5)}_{rstu} b^s_0 c^{s+} c^{r+} c^{t+} b^u_0 = 0. \tag{3.13}$$

After imposing the cyclic symmetry on the coefficients $X^{(1)}_{rstu}$ under the indexes r, s, t, u these equations can be solved [27] to give a solution given in the table 1. We can use the

Index combination	$X^{(1)}$	$X^{(2)}$	$X^{(3)}$	$X^{(5)}$
1231	1	-2	-1	1
1232	-1	0	1	-1
1233	0	0	0	1
1211	0	0	1	0
1212	-1	2	0	0
1213	-1	2	0	0
1221	1	-2	0	-4
1222	0	0	-1	1
1223	1	0	0	-1
1111	0	0	0	0
1112	-1	-2	1	0
1113	1	2	-1	0
1121	-2	-4	1	1
1122	-6	-12	-5	-4
1123	0	-2	-1	-1
1131	2	4	-1	-1
1132	0	2	1	1
1133	6	12	5	4
2131		0		
2132		2		
2133		0		
2111		0		
2112		2		
2113		0		
2121		-2		
2122		0		
2123		-2		

Table 1. Empty entries in the table mean that the corresponding value of the coefficient can be recovered from the ones given in the table using the cyclic property of indices (for example $X^{(i)}_{1231} = X^{(i)}_{2312} = X^{(i)}_{3123}$) and symmetry properties (3.8)

method of [71] to decompose our interacting Lagrangian in terms of irreducible Fronsdal modes. We will get a series of interacting Lagrangians for each mode which serve as a off-shell extension of the cubic vertex in [55]. Alternatively if we integrate out the auxiliary fields C and go on shell for the triplet fields as in [76, 77], we can derive from (3.6) an on-shell cubic vertex. The vertex for the highest spin mode from each triplet should be the same with the corresponding vertex for the irreducible HS fields of [55].

3.2 A cubic vertex for mixed symmetry fields

It is straightforward to generalize the results of the previous section to the case of "generalized triplets" i.e., for reducible representations of the Poincare group with mixed symmetry.

To this end one can replace indexes r, s, t with double indexes $\hat{r}, \hat{s}, \hat{t}$, where each index, say $\hat{r}$ has two values $\hat{r} = (r, \tilde{r})$. The index r again numerates three Hilbert spaces $r = 1, 2, 3$ while the index $\tilde{r}$ numerates different types of oscillators $\tilde{r} = 1, .., \infty$. The oscillators α^r_μ, c^r and b^r are replaced with $\alpha^{\hat{r}}_\mu$, $c^{\hat{r}}$ and $b^{\hat{r}}$. However, since one has only three different momenta p^r_μ and only three different ghost zero modes c^r_0 and b^r_0, one has $p^{\hat{1}}_\mu = p^{1,\tilde{1}}_\mu = p^{1,\tilde{2}}_\mu = \ldots p^{1,\tilde{n}}_\mu$, where $\tilde{n}$ is an arbitrary natural number. The BRST charge now reads as:

$$Q = c^i_0 l^i_0 + c^{\hat{i}+} l^{\hat{i}} + c^{\hat{i}} l^{\hat{i}+} - c^{\hat{i}+} c^{\hat{i}} b^i_0, \quad l^{\hat{i}\pm} = \alpha^{\hat{i}\pm}_\mu p^{\hat{i}}_\mu. \tag{3.14}$$

The symmetry properties of the coefficients X are again the same

$$X^{(1)}_{\hat{r}\hat{s}\hat{t}u} = X^{(1)}_{\hat{s}\hat{r}\hat{t}u}, \quad X^{(3)}_{\hat{r}\hat{s}\hat{t}u} = X^{(3)}_{\hat{s}\hat{r}\hat{t}u}, \quad X^{(5)}_{\hat{r}\hat{s}\hat{t}u} = -X^{(5)}_{\hat{t}\hat{s}\hat{r}u}. \tag{3.15}$$

The coefficients $X^{(1)}, X^{(2)}, X^{(3)}, X^{(5)}$ for a vertex, which describe the interaction between generalized triplets can be obtained from table 1. For example, the solution $X^{(1)}_{1231}$ corresponds to $X^{(1)}_{\hat{1},\hat{2},\hat{3},1} = X^{(1)}_{1\tilde{r},2\tilde{s},3\tilde{t},1}$ for $\tilde{r}, \tilde{s}, \tilde{t}, = 1, \ldots, \tilde{n}$ etc.

4 A quartic vertex

Let us consider the simple example given in (3.1) with the modification (3.3). For simplicity we consider the case for totally symmetric fields. As we mentioned before, although the gauge transformations are non-linear modifications of the free ones and the vertex cannot be removed via field redefinitions, it turns out that the algebra is abelian. One can verify using the formulas derived in [71] that the four-point function of any HS fields vanishes.

Therefore one can investigate another option, in particular instead of modifying the cubic vertex (3.1) using (3.3), we can add a quartic vertex to the Lagrangian and make it gauge invariant up to terms of order g^2. One can further try to continue this procedure iteratively to higher orders.

Let us consider this procedure for the solution (3.1). In order to construct the quartic interactions we take four Hilbert spaces and consider the Lagrangian

$$\begin{aligned} L = &\sum_{i=1}^{4} \int dc^i_0 \langle\Phi_i| \, Q_i \, |\Phi_i\rangle \\ &+ g\Big(\int dc^1_0 dc^2_0 dc^3_0 \langle\Phi_1|\langle\Phi_2|\langle\Phi_3||V_3\rangle + \int dc^1_0 dc^2_0 dc^4_0 \langle\Phi_1|\langle\Phi_2|\langle\Phi_4||V_3\rangle \\ &\quad + \int dc^2_0 dc^3_0 dc^4_0 \langle\Phi_2|\langle\Phi_3|\langle\Phi_4|V_3\rangle + \int dc^1_0 dc^3_0 dc^4_0 \langle\Phi_1|\langle\Phi_3|\langle\Phi_4||V_3\rangle + h.c\Big) \\ &+ g^2\left(\int dc^1_0 dc^2_0 dc^3_0 dc^4_0 \langle\Phi_1|\langle\Phi_2|\langle\Phi_3|\langle\Phi_4||V_4\rangle + h.c\right) \end{aligned} \tag{4.1}$$

and the nonlinear gauge transformations

$$\delta|\Phi_i\rangle = (\delta_0 + \delta_1 + \delta_2)|\Phi_i\rangle \tag{4.2}$$

JHEP11(2010)086

where

$$\delta_0|\Phi_i\rangle = Q_i|\Lambda_i\rangle \tag{4.3}$$

$$\delta_1|\Phi_i\rangle = -g\bigg(\int dc_0^{i+1}dc_0^{i+2}[(\langle\Phi_{i+1}|\langle\Lambda_{i+2}| + \langle\Phi_{i+2}|\langle\Lambda_{i+1}|)|V_3\rangle] \tag{4.4}$$
$$\int dc_0^{i+2}dc_0^{i+3}[(\langle\Phi_{i+2}|\langle\Lambda_{i+3}| + \langle\Phi_{i+3}|\langle\Lambda_{i+2}|)|V_3\rangle]+$$
$$\int dc_0^{i+1}dc_0^{i+3}[(\langle\Phi_{i+1}|\langle\Lambda_{i+3}| + \langle\Phi_{i+3}|\langle\Lambda_{i+1}|)|V_3\rangle]\bigg)$$

$$\delta_2|\Phi_i\rangle = (-1)^i g^2\int dc_0^{i+1}dc_0^{i+2}dc_0^{i+3}\bigg[\bigg(\langle\Phi_{i+1}|\langle\Phi_{i+2}|\langle\Lambda_{i+3}| + \langle\Phi_{i+1}|\langle\Phi_{i+3}|\langle\Lambda_{i+2}|+$$
$$\langle\Phi_{i+2}|\langle\Phi_{i+3}|\langle\Lambda_{i+1}|\bigg)|V_4\rangle\bigg] \tag{4.5}$$

As it was for the case of cubic interactions, the Lagrangian (4.1) is invariant up to zeroth order in the coupling constant i.e., under (4.3) since each separate BRST charge Q_i is nilpotent. Further, the Lagrangian is invariant up to the fist order in the coupling constant i.e., under (4.4) provided

$$(Q_i + Q_j + Q_k)|V_3\rangle = 0, \quad i \neq j \neq k. \tag{4.6}$$

Let us now investigate the conditions on the quartic vertex. Up to now our consideration has been completely general. In order to simplify further our analysis, let us take a specific expression (3.1) for the cubic vertex where cyclic symmetry between indexes i, j and k is assumed. We also take diagonal elements Y_{ii} and Z_{ii} equal to zero. From the cyclic property of the cubic vertex it follows that the quartic vertex obeys certain cyclic property as well. As can be seen from (4.1) the quartic vertex $|V_4\rangle$ should change sign under the cyclic permutation of indexes i, j, k, l

$$|V_4(1,2,3,4)\rangle = -|V_4(2,3,4,1)\rangle = |V_4(3,4,1,2)\rangle = -|V_4(4,1,2,3)\rangle \tag{4.7}$$

Our next step is to find a quartic vertex $|V_4\rangle$ from the requirement of cancellation of the terms of order g^2 in the variation of the Lagrangian. In particular, a variation of terms of the type

$$\tilde{L} = g\int dc_0^1 dc_0^2 dc_0^3\langle\Phi_1|\langle\Phi_2|\langle\Phi_3||V_3\rangle \tag{4.8}$$

under

$$\tilde{\delta}_1|\Phi_1\rangle = g\int dc_0^{2'}dc_0^{3'}[(\langle\Phi_{2'}|\langle\Lambda_{3'}| + \langle\Phi_{3'}|\langle\Lambda_{2'}|)|V_3(1,2',3')\rangle] \tag{4.9}$$

should be compensated by the variation of the free part of the Lagrangian under (4.5). The latter gives

$$(Q_1 + Q_2 + Q_3 + Q_4)\langle\Phi_{i+1}|\langle\Phi_{i+2}|\langle\Phi_{i+3}|\langle\Lambda_{i+4}||V_4\rangle + 3 \quad \text{permutations.} \tag{4.10}$$

In order to evaluate the former consider the variation of a typical term

$$\delta\tilde{L} = g^2\Big(\int dc_0^1 dc_0^2 dc_0^3 [\langle V_3(1,2,3)||\Phi_2\rangle|\Phi_3\rangle \int dc_0^{2'} dc_0^{3'} \langle\Lambda_{2'}|\langle\Phi_{3'}||V_3(1,2',3')\rangle = + \int dc_0^1 dc_0^2 dc_0^3 [\langle V_3(1,2,3)||\Phi_2\rangle|\Phi_3\rangle \int dc_0^{2'} dc_0^{3'} \langle\Lambda_{3'}|\langle\Phi_{2'}||V_3(1,2',3')\rangle\Big). \quad (4.11)$$

Let us integrate first over the ghost variables which have an index $i = 1$ evaluating explicitly the expression

$$\int dc_0^1 \langle 0_1|c_0^1 \exp(-Z_{ij}c^i b_0^j) \times \exp(Z_{mn}c^{+m}b_0^n)c_0^1|0_1\rangle = = Z_{a'1}c^{+a'} - Z_{a1}c^a, \quad a' = 2', 3', \quad a = 2, 3. \quad (4.12)$$

For the integration over the oscillator variable α_μ^{+1} we use

$$\langle 0_1|e^{Y_{1,i}\alpha_\mu^1 p_\mu^i} e^{Y_{1,i}\alpha_\mu^{+1} p_\mu^{i'}}|0_1\rangle = e^{Y_{1i'}Y_{1i}p_\mu^{i'}p_\mu^i} \quad (4.13)$$

where we have used the conservation of momentum $p_\mu^1 = -p_\mu^2 - p_\mu^3 = -p_\mu^{2'} - p_\mu^{3'}$. Collecting these results one arrives to the equation for the quartic vertex[3]

$$(Q_1 + Q_2 + Q_3 + Q_4)|V_4\rangle = 18(Z_{a'1}c^{+a'} + Z_{a1}c^{+a})e^{Y_{1i'}Y_{1i}p_\mu^{i'}p_\mu^i} e^M c_0^2 c_0^3 c_0^{2'} c_0^{3'}|0\rangle_{232'3'} \quad (4.14)$$

where

$$M = Y_{ai}\alpha_\mu^{+a}p_\mu^i + Y_{a'i'}\alpha_\mu^{+a'}p_\mu^{i'} + Z_{ab}c^{+a}b_0^b + Z_{a'b'}c^{+a'}b_0^{b'} \quad (4.15)$$

with $i/i' = 1, 2, 3/1, 2', 3'$, and $a, b/a'b' = 2, 3/2'3'$ and the right hand side of (4.14) can be written as a product of two cubic vertices with appropriate gluing of two Hilbert spaces between them [112]. This is totally analogous to the construction of the quartic vertex in OSFT by gluing two cubic vertices with a star product $V_4 \sim V_3 \star V_3$. An obvious ansatz for $|V_4\rangle$ is

$$|V_4\rangle = F(p)e^M c_0^2 c_0^3 c_0^{2'} c_0^{3'}|0\rangle_{232'3'} \quad (4.16)$$

where the unknown function $F(p)$ is determined from (4.14) to be

$$F(p) = \frac{18}{(p_\mu^2 + p_\mu^3)(p_\mu^{2'} + p_\mu^{3'})}. \quad (4.17)$$

Obviously the full solution is given by acting with the above given vertex on all non-cyclic permutations of the external states

$$\mathcal{L}_4 \sim \langle 1, 2, 3, 4|V_4\rangle_s + \langle 1, 3, 2, 4|V_4\rangle_u + \langle 1, 4, 2, 3|V_4\rangle_t \quad (4.18)$$

where each contribution has subscript indicating the massless pole on the corresponding kinematic variable which comes from the definition of $F(p)$ in (4.17) and we use the standard definitions for Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 + p_4)^2$ and $u = (p_1 + p_3)^2$. It turns out that the algebra of gauge transformations remains abelian in this case and

[3]We shall keep implicit the cyclic symmetry of the quartic vertex mentioned above.

one can check that the theory has vanishing four point functions. This is related to the fact that the on-shell expression of (3.1) gives, for a HS state coupling to the other two states, a current which is conserved identically and therefore should not deform the gauge algebra [42, 43]. Recently in an updated version of [55] the authors gave the completion of this vertex in the compensator formalism, with the modification implied by the ansatz (3.6) and (3.7). This is an extension of the non-local vertex (4.16) with the exponent M having additional terms due to the extra terms of ansatz (3.7), compared to those of the ansatz for a cubic vertex (3.1) which we used to derive the quartic vertex (4.16).

5 A short review of the BCFW method

All of the above attempts for constructing non-trivial interacting HS theories are plagued by several deficiencies. They deal with off-shell Lagrangians and therefore include non physical degrees of freedom which are eliminated only after proper gauge fixing. Also one can have various physically equivalent descriptions related by field redefinitions. Moreover we have seen that although it is relatively straightforward to derive cubic Lagrangians which are classically non-trivial it is not so obvious how one can classify which of these Lagrangians can really lead to non-trivial theories beyond the cubic level. In a sense the construction of the algebra through the commutators of gauge transformations is the correct way to classify which interactions are non-trivial. Nevertheless, as we explained above field redefinitions and gauge trivial degrees of freedom make these methods rather cumbersome to implement even at the quartic level, see i.e. [44–47]. Extending these methods beyond the quartic level is a very challenging task.

It is definitely desirable to have a method which deals directly with the physical degrees of freedom therefore allowing us to study the physical S-matrix of the theory. In any case all no-go theorems which forbid non-trivial HS interactions in 4-dimensions are formulated in the S-matrix language. Such a method has appeared recently [99, 100] (BCFW) based upon the twistor formulation of gauge theories of Witten [96]. This method allows one to construct, under certain assumptions, higher point functions given the cubic ones.

The key point of BCFW [100] is that tree level amplitudes constructed using Feynman rules are rational functions of external momenta. Analytic continuation of these momenta on the complex domain turns the amplitudes into meromorphic functions which can be constructed solely by their residues. Since the residues of scattering amplitudes are, due to unitarity, products of lower point on-shell amplitudes the final outcome is a set of powerful recursive relations.

The simplest complex deformation involves only two external particles whose momenta are shifted as

$$\hat{p}_i(z) = p_i - qz\,, \quad \hat{p}_j(z) = p_j + qz\,. \tag{5.1}$$

Here $z \in \mathbb{C}$ and to keep the on-shell condition we need $q \cdot p_i = q \cdot p_j = 0$ and $q^2 = 0$. In Minkowski space-time this is only possible for complex q. In particular, if the dimensionality of space-time is $d \geq 4$, we can choose a reference frame where the two external momenta

p_i and p_j are back to back with equal energy scaled to 1 [113]

$$p_i = (1, -1, 0, 0, \dots, 0), \quad p_j = (1, 1, 0, 0, \dots, 0), \quad q = (0, 0, 1, i, 0, \dots, 0)\ . \tag{5.2}$$

For gauge bosons the polarizations under a suitable gauge can be chosen as

$$\epsilon_i^+ = \epsilon_j^- = q, \quad \epsilon_i^- = \epsilon_j^+ = q^*, \quad \epsilon_T = (0, 0, 0, 0, \dots, 0, 1, 0, \dots, 0)\,, \tag{5.3}$$

where the $\pm$ superscripts correspond to the helicity of the states which should not to be confused with the usual light-cone notation for massless states. These vectors and the $D-4$ different polarizations ϵ_T form a basis in the transverse directions [113].

Under the deformation (5.1) the polarizations should become

$$\begin{aligned} &\hat{\epsilon}_i^+(z) = \hat{\epsilon}_j^-(z) = q, \quad \hat{\epsilon}_i^-(z) = q^* - zp_j, \quad \hat{\epsilon}_j^+(z) = q^* + zp_i, \\ &\hat{\epsilon}_T(z) = (0, 0, 0, 0, \dots, 0, 1, 0, \dots., 0) \end{aligned} \tag{5.4}$$

so that they remain orthogonal to the shifted momenta. In *four dimensions* we can use spinor representation of momenta and polarizations (for spin 1 states)

$$\begin{aligned} p^\mu &= \lambda^a (\sigma^\mu)_{a\dot{a}} \tilde{\lambda}^{\dot{a}} \\ \epsilon^+_{a\dot{a}} &= \frac{\mu_a \tilde{\lambda}_{\dot{a}}}{\langle \mu, \lambda \rangle}, \qquad \epsilon^-_{a\dot{a}} = \frac{\lambda_a \tilde{\mu}_{\dot{a}}}{[\tilde{\lambda}, \tilde{\mu}]} \\ \langle \mu, \lambda \rangle &\equiv \mu_a \lambda_b \epsilon^{ab} \qquad [\tilde{\lambda}, \tilde{\mu}] \equiv \tilde{\mu}_{\dot{a}} \tilde{\lambda}_{\dot{b}} \epsilon^{\dot{a}\dot{b}} \end{aligned} \tag{5.5}$$

with μ_a and $\tilde{\mu}_{\dot{a}}$ arbitrary reference spinors. Polarizations of Higher Spin states are given by products of the polarizations for spin 1

$$\epsilon^+_{a_1\dot{a}_1 \dots a_s \dot{a}_s} = \prod_{i=1}^{s} \epsilon^+_{a_i \dot{a}_i} \qquad \epsilon^-_{a_1\dot{a}_1 \dots a_s \dot{a}_s} = \prod_{i=1}^{s} \epsilon^-_{a_i \dot{a}_i} \tag{5.6}$$

The expressions above for the BCFW shifted momenta and polarizations correspond to the following shift on the spinors

$$\hat{\lambda}_a^{(i)}(z) = \lambda_a^{(i)} + z\lambda_a^{(j)}, \qquad \hat{\tilde{\lambda}}_{\dot{a}}^{(j)}(z) = \tilde{\lambda}_{\dot{a}}^{(j)} - z\tilde{\lambda}_{\dot{a}}^{(i)} \tag{5.7}$$

A general amplitude, which after the deformation becomes a meromorphic function $\mathcal{M}_n(z)$, will have simple poles for those values of z where the propagators of intermediate states go on shell (on the complex plane)

$$\frac{1}{P_J(z)^2} = \frac{1}{P_J(0)^2 - 2zq \cdot P_J}\ . \tag{5.8}$$

We denote by $P_J = \sum_{i \in J} p_i$ the total momentum of the intermediate state which connects the two sub-amplitudes in which $\mathcal{M}_n(z)$ factorizes. The undeformed amplitude can be computed using Cauchy's theorem:

$$\mathcal{M}_n(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{\mathcal{M}_n(z)}{z} dz = -\left\{ \sum \mathrm{Res}_{z=\text{finite}} + \mathrm{Res}_{z=\infty} \right\}\ . \tag{5.9}$$

As already stated, the residues at finite locations on the complex plane are necessarily, due to unitarity, products of lower point tree level amplitudes which are however computed at complex on-shell momenta. The residue at infinity can have a similar interpretation is some special cases [103, 104] but in general it cannot be written as product of lower point amplitudes. In most cases involving gauge bosons and/or gravitons there is an appropriate choice of the deformed external polarizations such that under a shift of the type (5.1), $\mathcal{M}_n(z)$ vanishes in the limit $z \to \infty$. In these cases the BCFW relation takes the simple form

$$\mathcal{M}_n(1,\ldots,n) = \sum_{r,h(r)} \sum_{k=2}^{n-2} \frac{\mathcal{M}_{k+1}(1,2,\ldots,\hat{i},\ldots,k,\hat{P}_r)\mathcal{M}_{n-k+1}(\hat{P}_r,k+1,\ldots,\hat{j},\ldots,n)}{(p_1+p_2+\ldots+p_k)^2}\,, \tag{5.10}$$

where the hat is used for the variables which are computed at the residue of the corresponding pole (5.8) and satisfy the physical state condition. Since we have made a complex deformation the hatted variables correspond to complex momenta, unlike the momenta of the rest of the external particles. The sum in r is over all different particles of the theory which can propagate in the intermediate channel and the sum in $h(r)$ is over their helicities.

In [108] a very interesting criterion was derived in order to classify, in four dimensions, which theories are constructible under BCFW deformations. The criterion has been stated explicitly for the four-point function and it is a necessary condition for a theory to have zero residue at infinity, i.e. to be constructible. Say we denote by $\mathcal{M}^{(i,j)}(z)$ the four-point function under deformation of particles i and j. Assume further on that the helicities h_2 and h_4 are negative while h_1 is positive. The criterion advocates that

$$\mathcal{M}_4^{(1,2)}(0) = \mathcal{M}_4^{(1,4)}(0) \tag{5.11}$$

This is highly non-trivial since the usual Feynman analysis construction of the four-point function requires adding diagrams from three possible channels while each BCFW deformation can use only two channels of exchanged particles. For $M^{(1,2)}(0)$ diagrams where particles 1 and 2 go to an intermediate state do not lead into poles on the complex plane i.e. $1/(p_1(z)+p_2(z))^2 = 1/(p_1+p_2)^2$. So in $M^{(1,2)}(z)$ only poles from the t, u channels on the complex z-plane will contribute. Viceversa for $M^{(1,4)}(0)$ only poles from the s, u channels on the complex-z plane will contribute. So the *crossing symmetry* condition (5.11) is a highly nontrivial constrain.

After this short review of the BCFW method we will proceed by considering some examples of theories of massless HS fields which are based on recent studies of the high energy limit of string theory [48, 56]. As was shown in [106, 107], tree level string amplitudes for appropriate regimes of the Mandelstam variables vanish when $z \to \infty$. Therefore one expects that these amplitudes also satisfy recursive relations similar to the field theoretic ones (5.10).The key point, as we will explain below, is that the methods employed in [106, 107] using the Pomeron technique of [114] suggest that the naive tensionless limit $\alpha' \to \infty$ does not spoil the good behavior of string amplitudes for large complex deformation of the momenta. Therefore one would expect that if a consistent high energy limit of string theory indeed exists then the theory would have to be BCFW constructible in the sense above. A study for some couplings was given in [108] but it was limited to self interactions and did not attempt to address either a complete theory or an example where all cubic couplings are given. We will try to shed some light on recently proposed theories of interacting HS and derive useful conclusions.

6 BCFW for Higher Spin theories

For any Lorentz invariant theory of massless particles in 4-dimensions it is straightforward to see that the on-shell cubic amplitudes vanish. Extending though some of the momenta in the complex plane we can easily derive that the most generic cubic amplitude is given by [108]

$$\mathcal{M}_3(\{\lambda^{(i)},\tilde{\lambda}^{(i)},h_i\}) = \kappa_H\langle 1,2\rangle^{d_3}\langle 2,3\rangle^{d_1}\langle 3,1\rangle^{d_2} + \kappa_A[1,2]^{-d_3}[2,3]^{-d_1}[3,1]^{-d_2}, \tag{6.1}$$

where we have defined $d_1 = h_1-h_2-h_3,\ d_2 = h_2-h_1-h_3,\ d_3 = h_3-h_1-h_2$ and the coupling constant k_H (k_A) is required to give the right dimension to the part of the amplitude which is holomorphic (antiholomorphic) in the spinor variables $\lambda^{(i)}$ ($\tilde{\lambda}^{(i)}$). Imposing that $\mathcal{M}_3$ has the correct physical behavior in the limit of real momenta. shows that if $d_1+d_2+d_3$, which is equal to $-h_1-h_2-h_3$, is negative (positive) then we must set $\kappa_H = 0$ ($\kappa_A = 0$) in order to avoid a singularity when any two (anti)holomorphic spinors become linearly dependent. The case when $h_1+h_2+h_3 = 0$ is more subtle since both pieces are allowed.

Under the conditions above one can show that $\mathcal{M}_4^{(1,2)}(0)$ is given by the expression [108]

$$\begin{aligned}\mathcal{M}_4^{(1,2)}(0) = &\sum_{h>\max(-(h_1+h_4),(h_2+h_3))}\left(\kappa^A_{1-h_1-h_4-h}\kappa^H_{1+h_2+h_3-h}\frac{(-P^2_{3,4})^h}{P^2_{1,4}}\left(\frac{[1,4][3,4]}{[1,3]}\right)^{h_4}\right.\\ &\left.\left(\frac{[1,3][1,4]}{[3,4]}\right)^{h_1}\left(\frac{\langle 3,4\rangle}{\langle 2,3\rangle\langle 2,4\rangle}\right)^{h_2}\left(\frac{\langle 2,4\rangle}{\langle 2,3\rangle\langle 3,4\rangle}\right)^{h_3}\right)\\ &+\sum_{h>\max(-(h_1+h_3),(h_2+h_4))}(4\leftrightarrow 3).\end{aligned} \tag{6.2}$$

where $P_{i,j} = p_i + p_j$. This is the final formula we will need to impose the constructibility criterion for the massless HS vertices we wish to examine. The subscript of the coupling constants denotes their mass dimension.

We will consider a theory with cubic couplings of two scalars and one HS field to study the $2 \to 2$ amplitude for scalars. Notice that usually in the BCFW method we consider external states which are gauge bosons. The reason is that the polarization tensors in (5.5), with polarization assignments as stated above, lead to factors $1/z$ and are important for the vanishing residue of $\mathcal{M}(z)$ at infinity. In our case though it is obvious that since the dimension of the couplings decreases with the spin, the behavior of individual Feynman diagrams becomes more and more divergent and therefore only subtle cancellations among Feynman diagrams can give a well behaved amplitude at infinite complex momenta. So we can work with the elementary process of scalar scattering which encodes all physics relevant to the theory.

We can write the cubic interaction for two scalars and one arbitrary HS triplet [71]. In order to compute the S-matrix elements for triplets one would need the propagator of the irreducible states they are composed of. This will introduce the normalization factors of [71]. In order to avoid technical complications which involve such factors and do not alter the physical properties we want to study, we will simplify our interacting Lagrangian and we will discuss a Lagrangian coupling of irreducible HS fields Ψ_h to two scalars [26]

$$\mathcal{L}^{00s}_{\text{int}} = \kappa^{1-h} N_h \, \frac{\Psi_h^{\mu_1 \ldots \mu_h} J^{1;2}_{h;\mu_1 \ldots \mu_h}}{h!} + \; h.c. \,, \tag{6.3}$$

where κ is a dimensionful constant needed in order to give the right dimension to the Lagrangian and we have assumed that the free action is normalized to give the properly normalized propagator for the fields with residue one. The interaction is proportional to the coefficients Y_{ij} of (3.2) but we shall not give the exact relation between them since our discussion will be more general than the specific solution. Moreover N_h is an undetermined constant which probably gets fixed once we have the fully consistent, interacting HS theory to all orders in the coupling constant. The currents are defined as

$$J^{1;2}_{h;\mu_1 \ldots \mu_h} = \sum_{r=0}^{h} \begin{pmatrix} h \\ r \end{pmatrix} \; (-1)^r \; (\partial^{\mu_1} \ldots \partial^{\mu_r} \phi_1) \; (\partial^{\mu_{r+1}} \ldots \partial^{\mu_h} \phi_2) \tag{6.4}$$

For example in the theory of [48] from the high energy limit of bosonic SFT one has $N_h = \sqrt{h!}$ for the top spin component of each triplet. Of course using the decomposition method in [71] the lower spin components of each triplet will have different normalizations dependent on the spin of the triplet they descend from. In principle in order to discuss on safe ground the high energy limit of SFT one should include all subleading Regge trajectories, mixed symmetry fields, while decomposing them into irreducible modes. For the purpose of our discussion we will concentrate into two main examples of couplings, which have been studied in [55, 75], with a simple spin dependence. The first example we will call *string theory* coupling has $N_h = \sqrt{h!}$ and it is the same as in [56] derived from string theory. The second one leads to spin independent coupling constants and we will call it *field theory* coupling. The normalization constant has a stronger growth with spin and is given by $N_h = h!$. This is unlike the softer *string theory* normalizations which lead into coupling constants which drop with increasing spin.

From the Lagrangian terms in (6.3) we can infer the three point on-shell amplitudes we will need for our test of the BCFW construction. We remind the reader that these three-point amplitudes make sense only for complex external momenta, since otherwise 4-dimensional Lorentz invariance requires that all on-shell three point amplitudes for massless states vanish. We can use directly (6.1) to infer that for particles 1 and 2 being scalars and the particle 3 being a spin h particle

$$\begin{aligned} M_3^{00+h}(\lambda_i, \tilde{\lambda}_i, h_i) &= k^A_{1-h}(h) \frac{[2,3]^h [3,1]^h}{[1,2]^h} \\ M_3^{00-h}(\lambda_i, \tilde{\lambda}_i, h_i) &= k^H_{1-h}(h) \frac{\langle 2,3\rangle^h \langle 3,1\rangle^h}{\langle 1,2\rangle^h} \end{aligned} \tag{6.5}$$

where the couplings of the two amplitudes are given by

$$k^A_{1-h}(h) = k^H_{1-h}(h) = \kappa^{1-h}\frac{N_h}{h!}. \tag{6.6}$$

Notice that the amplitudes must be symmetric under exchange of the two scalars. If the theory has only one scalar field or if we assume an abelian theory the spin h is restricted only to even values. We can assume couplings which have Chan-Paton factors for the scalars in such a way that we can include odd-spin couplings to our computations. In a way we will be computing colour ordered amplitudes. The total scattering amplitude will be given by the sum of the colour ordered terms with their corresponding color factors. Obviously if we go to the abelian case, i.e., when colour factors are trivial, only couplings to even HS fields will survive. As an example for the cubic amplitude there are two different color orderings given by exchange of particles 1 and 2 in (6.5). If we have a non-abelian theory they are nonequivalent but in the abelian case they will simply add up and since they have opposite signs for couplings to odd spin HS fields they will cancel. In what follows in order to avoid incorporating Chan-Paton factors in our notation, since it is an unnecessary complication, we will focus our discussion on the abelian case. Nevertheless since at several points we would like to compare our results with the explicit Feynman diagrams of [75], we will assume that there are at least two scalar fields or a complex one. In other words we will consider scalar fields which carry some charge. This will allow couplings of odd HS fields.

In order to derive the exact form of the couplings one would need to substitute (5.5), (5.6) in (6.3) where it turns out, as expected due to gauge invariance, that the dependence on the reference spinors μ_i drops out. Actually in this case we can easily write down a coupling for $h = 0$. Remember that this is the case where $h_1 + h_2 + h_3 = 0$ and both a holomorphic and antiholomorphic piece are allowed. In this case the situation is rather trivial and we can take unambiguously $h \to 0$ in (6.5) and we end up with a constant $M^{000} = N_0\kappa$.

Before we proceed with the study of BCFW recursion relations for these two kinds of couplings we would like to make a few comments for the string theory coupling. In string theory we expect the four point function to take the familiar Veneziano type amplitude behavior. This means that four point string amplitudes have Gamma function prefactors. A BCFW shift of the momenta results into shifts of two of the three Mandelstam kinematic invariants and naive Stirling approximation would give badly behaved amplitude for some region of complex infinity. Nevertheless the fact that these Gamma functions appear into Beta function combinations allows one to see that indeed the four point function is a well behaved polynomial of $1/z$ at infinity [105, 107]. For higher point functions it turns out it is more straightforward to use the Pomeron technique of [114] to study the behavior of amplitudes under BCFW deformations [106]. The typical behavior is

$$\mathcal{M}(z) \sim z^{n+1+\alpha' P_{12}^2} \tag{6.7}$$

where n is the level of the external string states which are being deformed and $P_{12} = p_1+p_2$. We see that choosing $n+1+\alpha' P_{12}^2 < 0$ we can conclude that all open bosonic string amplitudes are constructible. This might require an analytic continuation of the external states momenta outside the physical region but it is standard practice with string amplitude computations. This analytic continuation should not be confused with the complex momentum deformation. Moreover, the formulas we gave above in (6.2) contain no assumptions about reality of the undeformed spinors. Now it is obvious that taking $\alpha' \to \infty$ naively makes things only better.[4] But we know that the naive limit might lead to an non-interacting theory as well [86] so it might be a bit too naive to conclude anything more than good behavior of the amplitudes at infinity. It makes more sense to assume that if an interacting theory indeed exists in the high energy limit of string theory, then the string length reaches a critical value, $\alpha' \to \alpha'_{cr}$, rather than infinity. The Pomeron argument seems still valid so if indeed a consistent high energy limit of string theory does exist, we would expect that it passes the BCFW test (5.11). Finally a careful examination of the saddle point method, used to derive the Pomeron operators which led to the aforementioned behavior, does not show any pathology under the tensionless limit.

One can immediately see that there might be potential problems trying to apply the BCFW procedure to interacting massless HS theories like those discussed in the previous sections. The exponential type vertex in (4.16), unlike the Veneziano amplitude, does not have a good behavior through all complex infinity under BCFW deformations and we will have to discuss novel features like inclusion of Pomeron-like states in the theory in order address this problem.

In the next two subsections we will study the BCFW test of [108] for two potentially interesting cases: *string and field theory couplings*.

6.1 BCFW for field theory couplings

In this case we need to use the coupling constants $N_h = h!$. We will consider the scattering process $\phi(p_1)\ \phi(p_2) \to \phi(-p_3)\ \phi(-p_4)$ for two scalars with the same charge as in [75]. To this end we need to apply equation (6.2) using (6.5). A straightforward computation gives

$$\mathcal{M}_4^{(1,2)}(0) = \sum_{h \in\ \mathbb{Z}} \kappa^{2-2h} (-P_{3,4}^2)^h \left(\frac{1}{P_{1,4}^2} + \frac{1}{P_{1,3}^2} \right). \tag{6.9}$$

[4] A possible caveat to this argument is the following. The general form in (6.7) is

$$\mathcal{M}(z) \sim z^{n+1+\alpha' P_{12}^2}\ \mathcal{H}(\alpha', p_i, z) \tag{6.8}$$

We have two limits to take, $z \to \infty$ and $\alpha' \to \infty$. The function $\mathcal{H}(\alpha', p_i, z)$ is a polynomial in inverse powers of z, therefore the limit $z \to \infty$ should pose no problem. Since the leading term dependent to z , $z^{n+1+\alpha' P_{12}^2}$, can only behave better with $\alpha' \to \infty$ for suitable P_{12}^2 as explained above, we can assume taking first $z \to \infty$ and then the high energy limit. Even if $\mathcal{H}(\alpha', p_i, z)$ is divergent at the high energy limit this would not be a problem if we assume that the two limits commute. To put it another way the BCFW shift corresponds to taking a large hierarchy between kinematic variables of a given scattering amplitude i.e. $s \gg t$. Taking the high energy limit of the theory, which means $\alpha' s, \alpha' t \gg 1$, but keeping the original hierarchy between the kinematic variables should not be a problem and our arguments, about BCFW constructibility of the theory in the high energy limit, should hold.

Using the standard definitions for Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 + p_4)^2$ and $u = (p_1 + p_3)^2$ in $(+, -, -, -)$ signature we can write the result as

$$\mathcal{M}_4^{(1,2)}(0) = \kappa^2 \frac{1}{1 + \kappa^{-2} s} \left(\frac{-s}{tu} \right). \tag{6.10}$$

On the other hand we can exchange labels 2 and 4 in the expression (6.9). In this case only the u-channel (with an intermediate propagator $1/P_{1,3}^2$) contributes due to charge conservation. The s-channel amplitude, (with an intermediate propagator $1/P_{1,2}^2$,) is not allowed in this process but should be used in the process $\phi^*(p_1)\ \phi(p_2) \to \phi(-p_3)\ \phi^*(-p_4)$, where ϕ^* is the antiparticle of ϕ. Finally we get

$$\mathcal{M}_4^{(1,4)}(0) = \kappa^2 \frac{1}{1 + \kappa^{-2} t} \left(\frac{1}{u} \right). \tag{6.11}$$

We see immediately that the two results do not match. This tells us that either the theory is not constructible or there is some pathology in its definition. Computing the explicit Feynman diagrams as in [75] for *field theory couplings* of the form in (6.3) shows that indeed we have a function with a pole at finite radius on the complex kinematic plane and that the position of the pole depends on the kinematics. The t-channel of the four point amplitude has the form

$$\mathcal{M}_4^t \sim \frac{\kappa^2}{t} \left(\frac{1}{1 + \frac{\kappa^{-2}}{4}(\sqrt{s} + \sqrt{-u})^2} + \frac{1}{1 + \frac{\kappa^{-2}}{4}(\sqrt{s} - \sqrt{-u})^2} - 1 \right). \tag{6.12}$$

The full amplitude is given by summing the u-channel contributions as well. The full expression should be compared to the BCFW result. Looking at the expression above for the t-channel suggests two things. The first one is that the original series of massless exchanges, either in the Feynman method or the BCFW one, has a finite radius of convergence. Outside this radius the series is defined through analytic continuation just as $\sum_s z^s = \frac{1}{1-z}$. Thus it is fairly obvious to see that a BCFW shift as in (5.7) results in a vanishing amplitude at infinite complex deformation which means that the theory should be constructible. The second point is that we have a pole which depends on the kinematic variables. This suggests that there should be some extended object in the theory since the amplitude blows up for specific impact parameters and angles [54, 56]. The BCFW computation tells us the same thing through its failure of crossing symmetry under BCFW deformations which suggests that at some finite distance on the complex kinematic variables plane the massless theory has some ingredient missing in its definition.[5] Actually from the explicit expression in (6.12) we can be a bit more precise. The amplitude is a meromorphic function of the kinematic variables. All dependence on the square roots of the kinematic variables disappears once we add the two terms together. Therefore this function should be determined through its poles on the complex plane. Under a BCFW

[5]The final result in (6.10) has a massive pole outside the physical region i.e. for $s < 0$. In the physical region though it is a form factor for an apparent massless exchange of particles which depends on the center of mass energy for (6.10) and angle of scattering for (6.11).

shift (5.7) $\hat\lambda^{(1)}_a(z) = \lambda^{(1)}_a + z\lambda^{(2)}_a$, $\hat{\tilde\lambda}^{(2)}_{\dot a}(z) = \tilde\lambda^{(2)}_{\dot a} - z\tilde\lambda^{(1)}_{\dot a}$ the Mandelstam variables shift as

$$\hat s = s\,, \quad \hat t = t - z\langle 2,3\rangle[1,3]\,, \quad \hat u = u + z\langle 2,3\rangle[1,3]. \tag{6.13}$$

Then one can easily verify that there are three poles on the complex z-plane for the t-channel contribution and three for the u-channel respectively. The one which comes from the massless t-pole of (6.12) is the one whose $Res(\mathcal{M}(z)/z)$ reproduces the t-channel pole of (6.9). On this pole the amplitude factorizes into two scalar three point amplitudes which are however dressed with some form factors. The other two poles of (6.12) should come from the "extended object" poles which contribute the form factors of (6.10). To get the u-channel pole of (6.9) we will need to compute the residues of the u-channel amplitude $\mathcal{M}^u_4$ in a similar manner. We can repeat the exercise for scattering of real scalars. This will require, as we mentioned before, that in (6.9) we will sum over even spins only and the same for the Feynman diagrams computation in [75].

6.2 BCFW for string theory couplings

In this section we will try to repeat the analysis above for the *string theory* couplings. One should keep in mind that we are not certain about the exact nature of the four-point function at the high energy limit of string theory. The standard result of [86] gives a behavior which does not seem that it can be reproduced by a local field theory.[6] As pointed out in [56], for a Feynman diagram computations of the full 4-point function the subleading Regge trajectories of their vertex would be necessary. But this by itself would not be enough since a non-local quartic vertex would be required in order to allow one to reproduce the local and unitary result of [86]. Moreover, the high energy behavior of [86] is proportional to the fixed angle scattering of string amplitudes

$$\mathcal{M}_4 \sim e^{-\alpha' s\log\alpha' s - \alpha' t\log\alpha' t - \alpha' u\log\alpha' u}\;. \tag{6.14}$$

The result above is definitely divergent under any BCFW deformation. But on the other hand as we pointed out the Pomeron arguments suggest that under $\alpha' \to \infty$ high energy string amplitudes remain constructible. Instead of relying on the standard result of [86] we will take a more pedestrian approach and assume as mentioned before that the theory reaches a critical string length were the theory is interacting. We will then try to construct amplitudes via BCFW and apply the criterion (5.11). This will fail as we shall shortly see and we will suggest a possible explanation for the problem.

Lets first discuss the amplitude computed using current exchanges for massless HS states in [75] for the $t-channel$ for the process of two same charge scalar scattering as in subsection 6.1

$$\mathcal{M}^t_4 = -\frac{\kappa^2}{t}\left[2\,\exp\Big(-\frac{\kappa^{-2}}{4}(s-u)\Big)\,\cosh\Big(\frac{\kappa^{-2}}{2}\sqrt{-su}\Big) - 1\right]. \tag{6.15}$$

This amplitude has a structure very similar with the one of the vertex (4.16). In the Regge regime with $s \gg t$, which implies in the massless case $s \sim -u$, we easily derive

$$\mathcal{M}^t_4 \sim -\frac{\kappa^2}{t}\,\exp\big(-\kappa^{-2}\,s\big)\;. \tag{6.16}$$

[6] At least perturbatively non-local in the sense of [8].

For $s \sim -u \sim qz$ this corresponds to the BCFW shift (6.13) of momenta $p_1,\ p_4$ in (6.15) and certainly does not vanish for all z in complex infinity. But the statement for BCFW constructibility requires that the full amplitude vanishes at complex infinity and not individual Feynman diagrams. The u-channel contribution cannot cancel that of the t-channel at complex infinity as one can easily check. It is plausible though that the BCFW deformation of the corresponding quartic vertex (4.16) gives a contribution which cancels the divergence from the exchange diagram (6.16). The actual form of the quartic vertex contribution to the four point function is of the form

$$\mathcal{M}_4^{t;\,\mathrm{contact}} \sim \frac{18}{Yt}\ \exp\Big(-Y\,(s-u)\Big) \tag{6.17}$$

where we have assumed that all coupling constants of the theory are determined in terms of one scale $Y_{ij} \sim Y(\kappa)$. One can compute explicitly the quartic vertex contribution with the exact normalization and of course determine the exact form of $Y(\kappa)$. If we determine the exact form of Y and normalization of the contribution in (6.17) we might cancel, under BCFW shift, the problematic behavior in (6.16). This would of course imply that the theory is constructible. Nevertheless at this point we will not try to verify if such a cancellation is true since, as we shall see bellow, it does not seem possible to apply the criterion (5.11) successfully. This conflict between the apparent constructibility of the theory and failure of (5.11) seems to imply that we need to make further assumptions for the structure of the theory.

Before we proceed further it would be useful to discuss the role of mixed symmetry fields to the arguments above. We pointed out in section 4, that if one starts with the cubic vertex for the HS fields given in (3.1) and then adds the contribution of the quartic vertex (4.16) the full four point function vanishes. Presumably, performing similar computation the four point function for the theory where the cubic vertex is given by (3.6), could lead to a non-trivial four-point function. We would like to point out that the couplings of reducible mixed symmetry HS fields to two scalars are identical to those derived the vertex of (3.1) since the currents built from the two scalars are totally symmetric. Decomposing the reducible mixed symmetry fields[7] to their symmetric parts results into various HS symmetric fields with multiplicities dependent on the Young tableaux of the mixed symmetry field and moreover on the generalized triplet they descend from. So in order to make precise statements about the behavior of scattering amplitudes at infinite complex momenta, one would need to compute these couplings for all generalized triplets and show that the full amplitude constructed using Feynman diagrams vanishes for infinite complex momenta. Alternatively one could use the method of [113]. But unlike spin one and spin two gauge theories, where the Lagrangian analysis leads directly to a well-behaved theory at complex infinity, here it would be subtle cancellations between interactions of all HS modes which can grant a soft behavior. So we would need at least some knowledge of the full HS Lagrangian for all mixed symmetry fields.

In any case if such an analysis would be possible and the theory turns out to be well-behaved at complex infinite momenta, then one would expect that the theory with

[7]Irreducible mixed symmetry fields give zero coupling to a totally symmetric current.

such cubic vertex would pass the test (5.11) irrespective of the precise form of the quartic vertex. This is actually the power of BCFW consistency conditions, that if the test fails, either there is a missing ingredient of the theory or the BCFW deformed amplitude does not fall fast enough and there is a boundary contribution to the BCFW relations and the method is not applicable. We used the Pomeron analysis to argue that the boundary contribution is vanishing for the tensionless limit of string amplitudes. So we will use as working hypothesis that this holds true and the BCFW recursion relations should apply.

Let us look at the BCFW construction of the four point function for scalar fields for $N_h = \sqrt{h!}$.

$$\mathcal{M}_4^{(1,2)}(0) = \sum_{h\in\mathbb{Z}} \frac{\kappa^{2-2h}}{h!}(-P_{3,4}^2)^h \left(\frac{1}{P_{1,4}^2} + \frac{1}{P_{1,3}^2}\right) \tag{6.18}$$

$$\mathcal{M}_4^{(1,2)}(0) = \kappa^2 e^{-\kappa^{-2}s}\left(\frac{-s}{tu}\right) \tag{6.19}$$

On the other hand we can exchange label 2 and 4 and we get

$$\mathcal{M}_4^{(1,4)}(0) = \kappa^2 e^{-\kappa^{-2}t}\left(\frac{1}{u}\right) \tag{6.20}$$

where as mentioned in subsection 6.1 only the u-channel contributes for this scattering process. We immediately see a couple of problems. The first problem is that neither of the two expressions of the amplitude does allow for soft behaviour under all possible BCFW shifts. The second point is that the results in (6.19) and 6.20 would be rendered consistent if the exponential function $e^{-\kappa^{-2}s}$ could become i.e. $\left(\frac{1}{\kappa^{-2}s}\right)$, and similar for $e^{-\kappa^{-2}t}$, therefore fixing the problematic exponential behavior and allowing us to pass the test (5.11). It is obvious that such a thing is not possible since the expression in (6.18) is a Laurent series around $s = 0$ and cannot give a function with a pole at $s = 0$. Could we get a result like in the previous subsection with a pole at a some other location on the complex $s-$plane? As was pointed out in [56] and we commented above, only the totally symmetric part of mixed symmetry fields couples to the currents (6.4) and therefore leads to the same couplings as the totally symmetric fields. One expects of course that the normalization constants N_h of these terms will become dependent on the Regge trajectory each couplings originates from. So it is plausible that N_h might grow stronger with spin than $N_h \sim \sqrt{h!}$ of the totally symmetric fields leading to a behavior similar to (6.10) . This point deserves further investigation but the obstacle is still the knowledge of the full fledged HS theory. In any case, even if this is possible then the discussion of the previous subsection would apply and it would mean that one might have to include extended objects in the theory. Therefore the theory is not complete with the massless HS fields on their own. The behavior suggested in [86] seems beyond reach since we cannot expand (6.14) in terms of massless exchanges.

In order to get an idea of what might be the problem we should remind ourselves of some important features of the prototype case, the Veneziano amplitude. It is the beta

JHEP11(2010)086

function expansion of the Veneziano amplitude,

$$\mathcal{M}_4(s,t) = -\sum_{n=0}^{\infty} \frac{(\alpha(s)+1)(\alpha(s)+2)\dots(\alpha(s)+n)}{n!}\,\frac{1}{\alpha(t)-n} \tag{6.21}$$

where $\alpha(s) = \alpha' s + \alpha_0$ the Regge trajectory, which gives it the nice properties of dual models. The massive poles are responsible for the dual nature of string amplitudes. The result above in (6.21) can be equivalently expanded in s-channel poles with a simple exchange of s and t. In other words the expression above includes the s-channel contribution unlike field theory amplitudes where one needs to add separately the other channels in order to derive a consistent theory. In the tensionless case we have moved all massive poles of the above expression to become massless. It is obvious that this spoils the dual nature of string amplitudes. In order to obtain a consistent theory in the tensionless limit it seems necessary to include the collective contribution of the massive poles which were pushed to the massless level.

We have pointed out before that the asymptotic behavior of string amplitudes under BCFW deformations and their form in the Regge regime are directly connected. So based on the above comments regarding the Veneziano amplitudes it is instructive to look at its Regge limit to see what might be the missing ingredient in order to make the theory consistent under (5.11). In string theory we know that the Regge limit scattering is better described using Pomeron operators which effectively average the contributions of zeros and poles of a given amplitude as we take a kinematic variable much larger than the others. This is a very distinct behavior from the behavior of the amplitudes we studied in section 6.1. Pomerons have a structure with a stringy origin and amplitudes due to Pomeron exchanges present a diffusion like behavior in transverse position space [114].

Let us look first at the Pomeron states of string theory. They are deduced using the OPE of normal string vertex operators [114] and correspond to saddle point contributions of string amplitudes in the Regge regime. For bosonic open strings the OPE of two states with momenta p_1 and p_2 gives

$$\mathcal{V}_P \sim C_n\, \Pi(\alpha' p^2)\, e^{ip\cdot X} [q\cdot\partial X]^{1-\alpha' p^2} \tag{6.22}$$

where $p = p_1 + p_2$ and $q = p_1 - p_2$. The propagator of the Pomeron is

$$\Pi(\alpha' p^2) = \Gamma\left(\alpha' p^2 - 1\right)\ . \tag{6.23}$$

and C_n is an expression which depends on polarization and momenta of the level n states for which we consider the OPE. Pomerons in string theory are physical states but with a fractional oscillator number which renders them outside the normal Hilbert space.

$$\begin{aligned} L_0\mathcal{V}_P &= \alpha' p^2 + N - 1 = \alpha' p^2 + \left(1-\alpha' p^2\right) - 1 = 0\,, \\ L_1\mathcal{V}_P &\sim q\cdot p = (p_1+p_2)\cdot(p_1-p_2) = 0\ . \end{aligned} \tag{6.24}$$

Notice that in the tensionless limit these states remain in the spectrum.

The Veneziano scattering amplitude in the Regge regime $s \gg t$ can be computed either using the Pomeron vertex operator (6.22) or directly from the Veneziano amplitude [106, 114]

$$\begin{aligned}\mathcal{M}_4 &\sim \int_{-\infty}^{+\infty} dy|y|^{-2+\alpha' t}|1-y|^{-2+\alpha' s} \sim \\ &\sim 2\pi\,\Gamma(-1+\alpha' t)(\alpha' s)^{1-\alpha' t}\end{aligned} \tag{6.25}$$

Taking the tensionless limit $\alpha' \to \infty$ limit it implies

$$\mathcal{M}_4^t \sim \frac{1}{\sqrt{t}}\left(\frac{t}{s}\right)^{\kappa^{-2}t-1} e^{-\kappa^{-2}t} \tag{6.26}$$

where κ the critical string length we have assumed in order for our formulas to make sense. This should be compared to (6.16) where we see the marked difference of the two expressions: for the region $s \gg t$ where the two expressions apply one goes as $e^{-\kappa^{-2}s}$ and the other one has the typical Pomeron behavior $s^{\kappa^{-2}t-1}$. Our discussion suggests that massless current exchanges alone cannot lead into the behavior above. We propose that the expression in (6.26) is the appropriate asymptotic behavior, of the tensionless limit of the four point string amplitude, for BCFW deformations $s \sim -u \sim qz$.

Pomerons play role in the physics of scattering amplitudes in the Regge region of kinematic variables, as in the example described above. They appear as composite particles exchanged between oppositely highly boosted external states. It is plausible therefore that in the high energy limit we might have to consider additional states which appear in the intermediate channels of elementary particle scattering. Moreover it is possible that they do not represent asymptotic states of the theory. Pomerons in QCD represent coherent color-singlet objects built from gluons, which are exchanged between hadrons in the Regge regime. Here we suggest that there could be composite non-local objects in the theory of massless HS fields which contribute to the scattering of HS fields in the Regge regime. We emphasize that the true nature of the advocated objects away from the Regge regime is not understood at the moment. Moreover notice that the Pomeron in QCD like theories emerges from a perturbative resummation of Feynman diagrams while in the present context of the high energy limit of string theory emerges at tree level in the HS theory. So in this case they are not perturbation theory artifacts.

7 Conclusions

In the first part of this paper we demonstrated how one can build off –shell cubic and quartic interaction vertices for the string inspired systems which contain an infinite number of massless bosonic Higher Spin fields. We hope that these results could be useful for further investigation of consistency properties of interacting Higher Spin fields on Minkowski background. In particular, it would be interesting to perform explicit calculations using perturbation theory and study the consistency of higher order interactions.

Another interesting question is to consider an anti de Sitter space-time background. In particular to find similar vertices on an AdS background using the technique of [67–70] and

JHEP11(2010)086

to discuss a possible connection with the results of [10–14] and implications for AdS/CFT correspondence of these models.

In the second part of this paper we demonstrated how the BCFW method for consistency checking of the S-matrix for a given theory could shed some light to the construction of interacting Higher Spin theories. The key point is that failure of the consistency condition stated in [108], for theories which satisfy certain criteria under BCFW deformations, gives us some information on the possible modification of the theory in order to make it constructible. In essence it allows us to identify a possible missing ingredient in order to make consistent Higher Spin interactions in flat space-time. We showed that for two candidate cubic couplings the BCFW consistency condition fails.

In one case, the *field theory* coupling, we argued that the amplitude behaves well under BCFW deformation at infinite complex momenta and therefore the BCFW relations should hold. Failure of the condition (5.11) was attributed to additional states which might be needed in order to make the theory consistent with BCFW. Moreover the analysis suggested that these states could be extended objects.

In the second case, the *string theory* couplings, a naive analysis based on Feynman diagrams for massless current exchanges seems to imply that BCFW method should not be applicable. Nevertheless recent progress in string theory suggests [106, 107] that BCFW method applies to string theory amplitudes. There does not seem to be a difficulty in taking the tensionless limit of these analysis which in turn implies that massless Higher Spin theory derived from the high energy limit of string theory should also allow BCFW recursion relations. Since the central object in the asymptotic behavior for the tensile string analysis is the string Pomeron, we suggested that it is plausible that one should supplement the theory with non-local objects which have Pomeron like behavior in the Regge regime of Higher Spin amplitudes.

The upshot of both cases is that interacting Higher Spin theories in flat background with only point particle states although not improbable, might not be the ones related to the high energy limit of string theory. Although one can add perturbatively quartic and higher vertices in the Lagrangian, consistently with gauge invariance at each order, a non-trivial S-matrix, its analyticity properties and BCFW constructibility seem to imply that: one should consider Higher Spin theories in a larger frame which includes extended and/or non-local objects in their spectrum.

Acknowledgments

We are grateful for numerous discussions with X. Bekaert, P. Benincasa, F. Cachazo, A. Sagnotti and M. Taronna. The work of A. F. was supported by an INFN postdoctoral fellowship and partly supported by the Italian MIUR-PRIN contract 20075ATT78. The work of M.T. has been supported by a STFC rolling grant ST/G00062X/1.

Note added. When the present work was on its final stage for submission, an updated version of the paper [55] appeared in the archive which includes an analysis having an overlap with ideas of sections 3 and 4. In our paper use the triplet formulation for reducible

Higher Spin fields in order to extend off-shell the vertices proposed in the original version of [55]. On the other hand in [55] they have used the compensator formulation , which describes irreducible Higher Spin modes. The two results are closely related.

References

[1] M.A. Vasiliev, *Higher spin gauge theories in various dimensions*, *Fortsch. Phys.* **52** (2004) 702 [hep-th/0401177] [SPIRES].

[2] X. Bekaert, S. Cnockaert, C. Iazeolla and M.A. Vasiliev, *Nonlinear higher spin theories in various dimensions*, hep-th/0503128 [SPIRES].

[3] D. Sorokin, *Introduction to the classical theory of higher spins*, *AIP Conf. Proc.* **767** (2005) 172 [hep-th/0405069] [SPIRES].

[4] N. Bouatta, G. Compere and A. Sagnotti, *An introduction to free higher-spin fields*, hep-th/0409068 [SPIRES].

[5] X. Bekaert, I.L. Buchbinder, A. Pashnev and M. Tsulaia, *On higher spin theory: Strings, BRST, dimensional reductions*, *Class. Quant. Grav.* **21** (2004) S1457 [hep-th/0312252] [SPIRES].

[6] A. Campoleoni, *Metric-like Lagrangian Formulations for Higher-Spin Fields of Mixed Symmetry*, *Riv. Nuovo Cim.* **033** (2010) 123 [arXiv:0910.3155] [SPIRES].

[7] D. Francia, *On the relation between local and geometric Lagrangians for higher spins*, *J. Phys. Conf. Ser.* **222** (2010) 012002 [arXiv:1001.3854] [SPIRES].

[8] X. Bekaert, N. Boulanger and P. Sundell, *How higher-spin gravity surpasses the spin two barrier: no-go theorems versus yes-go examples*, arXiv:1007.0435 [SPIRES].

[9] A. Fotopoulos and M. Tsulaia, *Gauge Invariant Lagrangians for Free and Interacting Higher Spin Fields. A Review of the BRST formulation*, *Int. J. Mod. Phys.* **A 24** (2009) 1 [arXiv:0805.1346] [SPIRES].

[10] E.S. Fradkin and M.A. Vasiliev, *Cubic Interaction in Extended Theories of Massless Higher Spin Fields*, *Nucl. Phys.* **B 291** (1987) 141 [SPIRES].

[11] E.S. Fradkin and M.A. Vasiliev, *Candidate to the Role of Higher Spin Symmetry*, *Ann. Phys.* **177** (1987) 63 [SPIRES].

[12] M.A. Vasiliev, *Consistent equation for interacting gauge fields of all spins in (3+1)-dimensions*, *Phys. Lett.* **B 243** (1990) 378 [SPIRES].

[13] M.A. Vasiliev, *More on equations of motion for interacting massless fields of all spins in (3+1)-dimensions*, *Phys. Lett.* **B 285** (1992) 225 [SPIRES].

[14] M.A. Vasiliev, *Nonlinear equations for symmetric massless higher spin fields in (A)dS(d)*, *Phys. Lett.* **B 567** (2003) 139 [hep-th/0304049] [SPIRES].

[15] M.A. Vasiliev, *On Conformal, SL(4,R) and Sp(8,R) Symmetries of 4d Massless Fields*, *Nucl. Phys.* **B 793** (2008) 469 [arXiv:0707.1085] [SPIRES].

[16] M.A. Vasiliev, *Higher spin superalgebras in any dimension and their representations*, *JHEP* **12** (2004) 046 [hep-th/0404124] [SPIRES].

[17] E.D. Skvortsov and M.A. Vasiliev, *Geometric formulation for partially massless fields*, *Nucl. Phys.* **B 756** (2006) 117 [hep-th/0601095] [SPIRES].

JHEP11(2010)086

[18] D.P. Sorokin and M.A. Vasiliev, *Reducible higher-spin multiplets in flat and AdS spaces and their geometric frame-like formulation*, Nucl. Phys. **B 809** (2009) 110 [arXiv:0807.0206] [SPIRES].

[19] I.A. Bandos and J. Lukierski, *Tensorial central charges and new superparticle models with fundamental spinor coordinates*, Mod. Phys. Lett. **A 14** (1999) 1257 [hep-th/9811022] [SPIRES].

[20] I.A. Bandos, J. Lukierski and D.P. Sorokin, *Superparticle models with tensorial central charges*, Phys. Rev. **D 61** (2000) 045002 [hep-th/9904109] [SPIRES].

[21] C. Fronsdal, *Massless Fields with Integer Spin*, Phys. Rev. **D 18** (1978) 3624 [SPIRES].

[22] C. Fronsdal, *Singletons and Massless, Integral Spin Fields on de Sitter Space (Elementary Particles in a Curved Space. 7*, Phys. Rev. **D 20** (1979) 848 [SPIRES].

[23] S. Ouvry and J. Stern, *Gauge fields of any spin and symmetry*, Phys. Lett. **B 177** (1986) 335 [SPIRES].

[24] A.K.H. Bengtsson, I. Bengtsson and L. Brink, *Cubic interaction terms for arbitrarily extended supermultiplets*, Nucl. Phys. **B 227** (1983) 41 [SPIRES].

[25] F. Hussain, G. Thompson and P.D. Jarvis, *Massive and Massless Gauge Fields of Any Spin and Symmetry*, Phys. Lett. **B 216** (1989) 139 [SPIRES].

[26] F.A. Berends, G.J.H. Burgers and H. van Dam, *Explicit construction of conserved currents for massless fields of arbitrary spin*, Nucl. Phys. **B 271** (1986) 429 [SPIRES].

[27] A.K.H. Bengtsson, *BRST approach to interacting higher spin gauge fields*, Class. Quant. Grav. **5** (1988) 437 [SPIRES].

[28] A. Pashnev and M.M. Tsulaia, *Dimensional reduction and BRST approach to the description of aRegge trajectory*, Mod. Phys. Lett. **A 12** (1997) 861 [hep-th/9703010] [SPIRES].

[29] I.L. Buchbinder, A. Pashnev and M. Tsulaia, *Lagrangian formulation of the massless higher integer spin fields in the AdS background*, Phys. Lett. **B 523** (2001) 338 [hep-th/0109067] [SPIRES].

[30] I.L. Buchbinder, V.A. Krykhtin and A. Pashnev, *BRST approach to Lagrangian construction for fermionic massless higher spin fields*, Nucl. Phys. **B 711** (2005) 367 [hep-th/0410215] [SPIRES].

[31] I.L. Buchbinder, V.A. Krykhtin, L.L. Ryskina and H. Takata, *Gauge invariant Lagrangian construction for massive higher spin fermionic fields*, Phys. Lett. **B 641** (2006) 386 [hep-th/0603212] [SPIRES].

[32] I.L. Buchbinder, A.V. Galajinsky and V.A. Krykhtin, *Quartet unconstrained formulation for massless higher spin fields*, Nucl. Phys. **B 779** (2007) 155 [hep-th/0702161] [SPIRES].

[33] I.L. Buchbinder, A.V. Galajinsky and V.A. Krykhtin, *Quartet unconstrained formulation for massless higher spin fields*, Nucl. Phys. **B 779** (2007) 155 [hep-th/0702161] [SPIRES].

[34] I.L. Buchbinder, V.A. Krykhtin and A.A. Reshetnyak, *BRST approach to Lagrangian construction for fermionic higher spin fields in AdS space*, Nucl. Phys. **B 787** (2007) 211 [hep-th/0703049] [SPIRES].

[35] K.B. Alkalaev and M. Grigoriev, *Unified BRST description of AdS gauge fields*, Nucl. Phys. **B 835** (2010) 197 [arXiv:0910.2690] [SPIRES].

[36] A.P. Isaev, S.O. Krivonos and O.V. Ogievetsky, *BRST operators for W algebras*, J. Math. Phys. **49** (2008) 073512 [arXiv:0802.3781] [SPIRES].

JHEP11(2010)086

[37] R.R. Metsaev, *Arbitrary spin massless bosonic fields in d-dimensional anti-de Sitter space*, hep-th/9810231 [SPIRES].

[38] R.R. Metsaev, *Arbitrary spin massless bosonic fields in d-dimensional anti-de Sitter space*, hep-th/9810231 [SPIRES].

[39] Y.M. Zinoviev, *Massive $N = 1$ supermultiplets with arbitrary superspins*, *Nucl. Phys.* **B 785** (2007) 98 [arXiv:0704.1535] [SPIRES].

[40] P. de Medeiros and C. Hull, *Exotic tensor gauge theory and duality*, *Commun. Math. Phys.* **235** (2003) 255 [hep-th/0208155] [SPIRES].

[41] P. de Medeiros and C. Hull, *Geometric second order field equations for general tensor gauge fields*, *JHEP* **05** (2003) 019 [hep-th/0303036] [SPIRES].

[42] R.R. Metsaev, *Cubic interaction vertices for fermionic and bosonic arbitrary spin fields*, arXiv:0712.3526 [SPIRES].

[43] R.R. Metsaev, *Cubic interaction vertices for massive and massless higher spin fields*, *Nucl. Phys.* **B 759** (2006) 147 [hep-th/0512342] [SPIRES].

[44] X. Bekaert, N. Boulanger and S. Cnockaert, *Spin three gauge theory revisited*, *JHEP* **01** (2006) 052 [hep-th/0508048] [SPIRES].

[45] N. Boulanger, S. Leclercq and S. Cnockaert, *Parity violating vertices for spin-3 gauge fields*, *Phys. Rev.* **D 73** (2006) 065019 [hep-th/0509118] [SPIRES].

[46] N. Boulanger and S. Leclercq, *Consistent couplings between spin-2 and spin-3 massless fields*, *JHEP* **11** (2006) 034 [hep-th/0609221] [SPIRES].

[47] N. Boulanger, S. Leclercq and P. Sundell, *On The Uniqueness of Minimal Coupling in Higher-Spin Gauge Theory*, *JHEP* **08** (2008) 056 [arXiv:0805.2764] [SPIRES].

[48] A. Fotopoulos and M. Tsulaia, *Interacting Higher Spins and the High Energy Limit of the Bosonic String*, *Phys. Rev.* **D 76** (2007) 025014 [arXiv:0705.2939] [SPIRES].

[49] Y.M. Zinoviev, *On spin 3 interacting with gravity*, *Class. Quant. Grav.* **26** (2009) 035022 [arXiv:0805.2226] [SPIRES].

[50] Y.M. Zinoviev, *Spin 3 cubic vertices in a frame-like formalism*, *JHEP* **08** (2010) 084 [arXiv:1007.0158] [SPIRES].

[51] R. Manvelyan, K. Mkrtchyan and W. Rühl, *General trilinear interaction for arbitrary even higher spin gauge fields*, *Nucl. Phys.* **B 836** (2010) 204 [arXiv:1003.2877] [SPIRES].

[52] R. Manvelyan, K. Mkrtchyan and W. Ruehl, *Direct construction of a cubic selfinteraction for higher spin gauge fields*, arXiv:1002.1358 [SPIRES].

[53] R. Manvelyan, K. Mkrtchyan and W. Rühl, *Off-shell construction of some trilinear higher spin gauge field interactions*, *Nucl. Phys.* **B 826** (2010) 1 [arXiv:0903.0243] [SPIRES].

[54] A. Fotopoulos, unpublished.

[55] A. Sagnotti and M. Taronna, *String Lessons for Higher-Spin Interactions*, *Nucl. Phys.* **B 842** (2011) 299 [arXiv:1006.5242] [SPIRES].

[56] M. Taronna, *Higher Spins and String Interactions*, arXiv:1005.3061 [SPIRES].

[57] D. Polyakov, *Gravitational Couplings of Higher Spins from String Theory*, *Int. J. Mod. Phys.* **A 25** (2010) 4623 [arXiv:1005.5512] [SPIRES].

[58] D. Polyakov, *Interactions of Massless Higher Spin Fields From String Theory*, *Phys. Rev.* **D 82** (2010) 066005 [arXiv:0910.5338] [SPIRES].

[59] F. Bastianelli and R. Bonezzi, *U(N) spinning particles and higher spin equations on complex manifolds*, *JHEP* **03** (2009) 063 [arXiv:0901.2311] [SPIRES].

[60] F. Bastianelli, O. Corradini and E. Latini, *Spinning particles and higher spin fields on (A)dS backgrounds*, *JHEP* **11** (2008) 054 [arXiv:0810.0188] [SPIRES].

[61] O. Corradini, *Half-integer Higher Spin Fields in (A)dS from Spinning Particle Models*, *JHEP* **09** (2010) 113 [arXiv:1006.4452] [SPIRES].

[62] S. Deser and A. Waldron, *Partial masslessness of higher spins in (A)dS*, *Nucl. Phys.* **B 607** (2001) 577 [hep-th/0103198] [SPIRES].

[63] A.R. Gover, A. Shaukat and A. Waldron, *Weyl Invariance and the Origins of Mass*, *Phys. Lett.* **B 675** (2009) 93 [arXiv:0812.3364] [SPIRES].

[64] D. Cherney, E. Latini and A. Waldron, *Generalized Einstein Operator Generating Functions*, *Phys. Lett.* **B 682** (2010) 472 [arXiv:0909.4578] [SPIRES].

[65] R. Marnelius, *Lagrangian higher spin field theories from the O(N) extended supersymmetric particle*, arXiv:0906.2084 [SPIRES].

[66] A. Sagnotti and M. Tsulaia, *On higher spins and the tensionless limit of string theory*, *Nucl. Phys.* **B 682** (2004) 83 [hep-th/0311257] [SPIRES].

[67] A. Fotopoulos, K.L. Panigrahi and M. Tsulaia, *Lagrangian formulation of higher spin theories on AdS space*, *Phys. Rev.* **D 74** (2006) 085029 [hep-th/0607248] [SPIRES].

[68] I.L. Buchbinder, A. Fotopoulos, A.C. Petkou and M. Tsulaia, *Constructing the cubic interaction vertex of higher spin gauge fields*, *Phys. Rev.* **D 74** (2006) 105018 [hep-th/0609082] [SPIRES].

[69] A. Fotopoulos, N. Irges, A.C. Petkou and M. Tsulaia, *Higher-Spin Gauge Fields Interacting with Scalars: The Lagrangian Cubic Vertex*, *JHEP* **10** (2007) 021 [arXiv:0708.1399] [SPIRES].

[70] A. Fotopoulos and M. Tsulaia, *Current Exchanges for Reducible Higher Spin Modes on AdS*, arXiv:1007.0747 [SPIRES].

[71] A. Fotopoulos and M. Tsulaia, *Current Exchanges for Reducible Higher Spin Multiplets and Gauge Fixing*, *JHEP* **10** (2009) 050 [arXiv:0907.4061] [SPIRES].

[72] D. Francia, J. Mourad and A. Sagnotti, *Current exchanges and unconstrained higher spins*, *Nucl. Phys.* **B 773** (2007) 203 [hep-th/0701163] [SPIRES].

[73] D. Francia, J. Mourad and A. Sagnotti, *(A)dS exchanges and partially-massless higher spins*, *Nucl. Phys.* **B 804** (2008) 383 [arXiv:0803.3832] [SPIRES].

[74] A. Sagnotti, *Higher Spins and Current Exchanges*, arXiv:1002.3388 [SPIRES].

[75] X. Bekaert, E. Joung and J. Mourad, *On higher spin interactions with matter*, *JHEP* **05** (2009) 126 [arXiv:0903.3338] [SPIRES].

[76] D. Francia and A. Sagnotti, *On the geometry of higher-spin gauge fields*, *Class. Quant. Grav.* **20** (2003) S473 [hep-th/0212185] [SPIRES].

[77] D. Francia, *String theory triplets and higher-spin curvatures*, *Phys. Lett.* **B 690** (2010) 90 [arXiv:1001.5003] [SPIRES].

JHEP11(2010)086

[78] E. Sezgin and P. Sundell, *Massless higher spins and holography*, Nucl. Phys. **B 644** (2002) 303 [*Erratum ibid.* **B 660** (2003) 403] [hep-th/0205131] [SPIRES].

[79] I.R. Klebanov and A.M. Polyakov, *AdS dual of the critical O(N) vector model*, Phys. Lett. **B 550** (2002) 213 [hep-th/0210114] [SPIRES].

[80] A.C. Petkou, *Evaluating the AdS dual of the critical O(N) vector model*, JHEP **03** (2003) 049 [hep-th/0302063] [SPIRES].

[81] R.G. Leigh and A.C. Petkou, *Holography of the $N = 1$ higher-spin theory on AdS_4*, JHEP **06** (2003) 011 [hep-th/0304217] [SPIRES].

[82] M. Bianchi, J.F. Morales and H. Samtleben, *On stringy $AdS_5 \times S^5$ and higher spin holography*, JHEP **07** (2003) 062 [hep-th/0305052] [SPIRES].

[83] P. Haggi-Mani and B. Sundborg, *Free large-N supersymmetric Yang-Mills theory as a string theory*, JHEP **04** (2000) 031 [hep-th/0002189] [SPIRES].

[84] S. Giombi and X. Yin, *Higher Spin Gauge Theory and Holography: The Three-Point Functions*, JHEP **09** (2010) 115 [arXiv:0912.3462] [SPIRES].

[85] S. Giombi and X. Yin, *Higher Spins in AdS and Twistorial Holography*, arXiv:1004.3736 [SPIRES].

[86] D.J. Gross and P.F. Mende, *String Theory Beyond the Planck Scale*, Nucl. Phys. **B 303** (1988) 407 [SPIRES].

[87] N. Moeller and P.C. West, *Arbitrary four string scattering at high energy and fixed angle*, Nucl. Phys. **B 729** (2005) 1 [hep-th/0507152] [SPIRES].

[88] G. Bonelli, *On the tensionless limit of bosonic strings, infinite symmetries and higher spins*, Nucl. Phys. **B 669** (2003) 159 [hep-th/0305155] [SPIRES].

[89] U. Lindström and M. Zabzine, *Tensionless strings, WZW models at critical level and massless higher spin fields*, Phys. Lett. **B 584** (2004) 178 [hep-th/0305098] [SPIRES].

[90] H. Gustafsson, U. Lindström, P. Saltsidis, B. Sundborg and R. van Unge, *Hamiltonian BRST quantization of the conformal string*, Nucl. Phys. **B 440** (1995) 495 [hep-th/9410143] [SPIRES].

[91] U. Lindström and M. Roček, *$D = 2$ null superspaces*, Phys. Lett. **B 271** (1991) 79 [SPIRES].

[92] U. Lindström, B. Sundborg and G. Theodoridis, *The Zero tension limit of the spinning string*, Phys. Lett. **B 258** (1991) 331 [SPIRES].

[93] U. Lindström, B. Sundborg and G. Theodoridis, *The Zero tension limit of the superstring*, Phys. Lett. **B 253** (1991) 319 [SPIRES].

[94] A. Karlhede and U. Lindström, *The classical bosonic string in the zero tension limit*, Class. Quant. Grav. **3** (1986) L73 [SPIRES].

[95] I.G. Koh and S. Ouvry, *Interacting gauge fields of any spin and symmetry*, Phys. Lett. **B 179** (1986) 115 [*Erratum ibid.* **B 183** (1987) 434 E] [SPIRES].

[96] E. Witten, *Perturbative gauge theory as a string theory in twistor space*, Commun. Math. Phys. **252** (2004) 189 [hep-th/0312171] [SPIRES].

[97] F. Cachazo, P. Svrček and E. Witten, *MHV vertices and tree amplitudes in gauge theory*, JHEP **09** (2004) 006 [hep-th/0403047] [SPIRES].

[98] S.J. Parke and T.R. Taylor, *An Amplitude for n Gluon Scattering*, Phys. Rev. Lett. **56** (1986) 2459 [SPIRES].

JHEP11(2010)086

[99] R. Britto, F. Cachazo and B. Feng, *New Recursion Relations for Tree Amplitudes of Gluons*, *Nucl. Phys.* **B 715** (2005) 499 [hep-th/0412308] [SPIRES].

[100] R. Britto, F. Cachazo, B. Feng and E. Witten, *Direct Proof Of Tree-Level Recursion Relation In Yang-Mills Theory*, *Phys. Rev. Lett.* **94** (2005) 181602 [hep-th/0501052] [SPIRES].

[101] F. Cachazo and P. Svrček, *Lectures on twistor strings and perturbative Yang-Mills theory*, PoS(RTN2005)004 (2005) [hep-th/0504194] [SPIRES].

[102] K. Risager, *A direct proof of the CSW rules*, *JHEP* **12** (2005) 003 [hep-th/0508206] [SPIRES].

[103] B. Feng, J. Wang, Y. Wang and Z. Zhang, *BCFW Recursion Relation with Nonzero Boundary Contribution*, *JHEP* **01** (2010) 019 [arXiv:0911.0301] [SPIRES].

[104] B. Feng and C.-Y. Liu, *A note on the boundary contribution with bad deformation in gauge theory*, *JHEP* **07** (2010) 093 [arXiv:1004.1282] [SPIRES].

[105] R. Boels, K.J. Larsen, N.A. Obers and M. Vonk, *MHV, CSW and BCFW: field theory structures in string theory amplitudes*, *JHEP* **11** (2008) 015 [arXiv:0808.2598] [SPIRES].

[106] C. Cheung, D. O'Connell and B. Wecht, *BCFW Recursion Relations and String Theory*, *JHEP* **09** (2010) 052 [arXiv:1002.4674] [SPIRES].

[107] R.H. Boels, D. Marmiroli and N.A. Obers, *On-shell Recursion in String Theory*, *JHEP* **10** (2010) 034 [arXiv:1002.5029] [SPIRES].

[108] P. Benincasa and F. Cachazo, *Consistency Conditions on the S-matrix of Massless Particles*, arXiv:0705.4305 [SPIRES].

[109] D.J. Gross and A. Jevicki, *Operator Formulation of Interacting String Field Theory*, *Nucl. Phys.* **B 283** (1987) 1 [SPIRES].

[110] D.J. Gross and A. Jevicki, *Operator Formulation of Interacting String Field Theory. 2*, *Nucl. Phys.* **B 287** (1987) 225 [SPIRES].

[111] A. Neveu and P.C. West, *Symmetries of the Interacting Gauge Covariant Bosonic String*, *Nucl. Phys.* **B 278** (1986) 601 [SPIRES].

[112] A.K.H. Bengtsson, *Structure of Higher Spin Gauge Interactions*, *J. Math. Phys.* **48** (2007) 072302 [hep-th/0611067] [SPIRES].

[113] N. Arkani-Hamed and J. Kaplan, *On Tree Amplitudes in Gauge Theory and Gravity*, *JHEP* **04** (2008) 076 [arXiv:0801.2385] [SPIRES].

[114] R.C. Brower, J. Polchinski, M.J. Strassler and C.-I. Tan, *The Pomeron and Gauge/String Duality*, *JHEP* **12** (2007) 005 [hep-th/0603115] [SPIRES].

JHEP11(2010)086