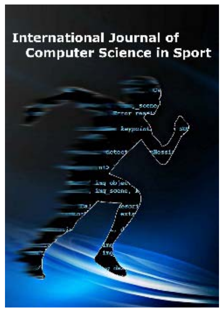
International Journal of Computer Science in Sport

Volume 17, Issue 1, 2018

Journal homepage: http://iacss.org/index.php?id=30

sciendo

DOI: 10.2478/ijcss-2018-0003



Some New ‘Short Games’ Within a Set of Tennis

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Abstract

Recently there has been an interest in developing tennis scoring systems that involve playing a fewer number of points on average. In devising such ‘shorter’ tennis scoring systems, it would be ideal for them to also have the following four characteristics: A smaller standard deviation of duration, a similar value for the probability that the better player wins, an increased efficiency, and a greater average excitement per point played. Thus, in total there are five considerations when devising such new scoring systems. Quite often in this type of study a scoring system that is ‘better’ with regard to one of these characteristics is ‘worse’ with regard to another (or others). In this paper we outline some new tennis scoring systems that have improvements in all (or almost all) of these five characteristics. We identify 3 or 4 different game structures that could be useful for tournaments. A common thread in the approach taken is the elimination of unimportant and unexciting points within the game structure. The choice of which is the most appropriate new format for a particular tournament would depend amongst other things on the planned reduction in the expected set duration.

KEYWORDS: ADVANTAGE GAME, NO AD GAME, EFFICIENCY, EXCITEMENT, IMPORTANCE

Introduction

There is quite a history in using mathematics and probability to study the characteristics of scoring systems in sport. A few early examples in tennis and squash are by ApSimon(1957), Carter and Crews (1974), Clarke and Norman (1979), Schutz (1970), Schutz and Kinsey (1977), Fischer (1980), Miles (1984), Pollard (1983, 1986). Some more recent examples are by Barnett, Brown and Pollard (2007), Pollard and Noble (2003, 2004).

There has been a practical interest in different tennis scoring systems for some decades. For example, the 12-point tiebreak (TB) game was introduced at Wimbledon in 1972. The no ad game has been an approved scoring system within the Rules of Tennis for some years. Even more recently a (short) set with the winner winning 4-0, 4-1, 4-2 or 4-3 after a TB game at 3-3, and possibly using no ad games, has been used in some (lower rated) tournaments and televised exhibition matches.The main reasons for the introduction of these changes has been to reduce the average time taken to play a match, and to reduce the likelihood of 'long' matches.

Similar 'short versions' of scoring have been developed in other sports. For example, in squash by not using the 'hand-out' system of service exchange, and in cricket with one day internationals and Twenty20 versions of the game. A six hole version of golf has also been trialed recently. Thus, it is clear that there is a demand for new versions of various sports so that they can more reliably be completed in less time.

Given that shorter tennis sets are presently of interest, the purpose of this paper is to consider how changing the method of scoring within each game of tennis but leaving the game structure within each set structure just as it is at present, might affect the duration and other important characteristics of a set of tennis.

Methods

In this paper we consider the effect of using different types of games within a tiebreak set. At present there are advantage games and no ad games that are available and used, but others can be devised. Given the interest in playing sets that are 'shorter', we ask the question: Is there a new type of 'shorter' game that in some sense is 'best overall' (or the best compromise) when used within the present TB set for, say, men's tennis? Further, how would we evaluate such a claim?

Several measures or characteristics such as (i) the average number of points played in a set, (ii) the probability that the better player wins the set, (iii) the efficiency of the set scoring system, and (iv) the standard deviation (SD) of the number of points played in the set, are typically used in making such comparisons or evaluations of scoring systems. A scoring system that has a very high value for the probability that the better player wins typically has a very high value for the expected value and variance of the number of points played. There is a need for a compromise or balance between these measures. A 'good' scoring system for a set of tennis has an appropriate number of points played on average, an appropriate value for the probability that the better player wins, a relatively good efficiency, and a relatively small SD of the number of points played.

Morris (1977) defined the *importance* of a point within a game of tennis as the difference between the probability player A wins the game given he wins the point minus the probability that he wins the game given he loses that point. In a very elegant paper Miles (1984) defined the *efficiency* of a tennis scoring system (which can be viewed as a statistical test of the hypothesis of the equality of two binomial probabilities). The efficiency of a tennis scoring system is given by

$$2(P - Q)\ln(P/Q)\,/(\mu(pA - pB)\ln\left(\frac{\text{pAqB}}{\text{pBqA}}\right)) ,$$

where μ is the mean number of points played, P is the probability that the better player wins, Q = 1 – P, p_A is the probability the better player wins a point when serving, and p_B is the probability the other player wins a point when serving. q_A =1-p_A and q_B = 1-p_B.

It can be seen that given two scoring systems with the same value of P, the more efficient one is the one with the smaller expected number of points played. Pollard (1992, p. 277) showed that the efficiency of a tennis scoring system increased as the variance of the importances of the points decreased.

Pollard (2017a) defined the *excitement* of a point within a scoring system as the expected value of the absolute size of the change in a player's probability of winning as a result of that point being played. He noted the relationship between the excitement of a point and the importance of that point… viz, the excitement of a point, Ex, is the importance of that point, I, multiplied by 2*p*(1 - p) where p is player A's probability of winning that point. Thus, the excitement of a point is somewhat similar to, but nevertheless different from, the importance of that point. Thus, we can add a fifth measure or characteristic (in addition to the four above) when making comparisons of scoring systems, and that measure is the average excitement of a point played in the set.

It is quite possibly of interest to some readers that Pollard (2017b) noted the relationship of excitement, importance and entropy to the efficiency of some statistical tests for the equality of two binomial probabilities (which is in fact the tennis context, as noted by Miles(1984)). This paper also includes several new and quite general scoring systems theorems.

Earlier, Pollard (2002) noted that when player A's probability of winning a point was 0.6219 (a representative value in the range of appropriate values for men's singles tennis), the importances of the points 30-40 (or adR), 15-40, 15-30 and 30-30 (or deuce) within the advantage game have values of 0.7302, 0.4541, 0.4477 and 0.4439 respectively. These are the four most important and most exciting points within such an advantage game. The five least important and least exciting points within this game are 40-0, 40-15, 30-0, 15-0 and 30-15 and these have importances of 0.0386, 0.1020, 0.1114, 0.1877 and 0.2312 respectively. It is clear that 40-0, for example, is indeed a very unimportant and unexciting point relative to the most exciting points within the advantage game. It is clear that 40-0 is also a very unimportant and unexciting point within the other types of game scoring systems. ***So why bother playing such a point?*** One can clearly reduce the expected duration of a set, increase the average excitement and importance of the points, and increase the efficiency of the set by not playing such points. This idea leads naturally to the fifth type of game listed below where the two unimportant and unexciting points 40-0 and 40-15 are removed.

The 'best' of a number of scoring systems could be considered to be the one that in some sense 'comes out best overall' across the five measures or characteristics mentioned above. There could be more than just one system that is considered 'best'. Thus, in this paper we consider different types of possible game scoring systems, whilst leaving the remainder of the tiebreak set unchanged.

The six types of games considered in this paper are,

1. the normal or classical advantage game that has been used since tennis began. The winner of this game is the first person to win at least 4 points and win at least 2

more points than the opponent. This game is the natural one against which all other alternatives could be compared.

2. the no ad game which is an approved game within the Rules of Tennis and has been used in recent times, particularly in doubles and in exhibition singles events. This game clearly has a smaller mean and SD of duration than the advantage game in 1. above. Thus, there are fewer 'long' sets when using the no ad game. However, when using this game the probability that the better player wins the set is reduced a little compared to when using the advantage game.
3. the 30-30 advantage game. The winner of this game is the first person to win at least 3 points and win at least 2 points more than the opponent. Thus, the server can win 40-0, 40-15, or win after 'deuce' is reached at 30-30. Correspondingly, the server can lose 0-40, 15-40, or lose after 'deuce' is reached at 30-30. Thus, this game is very similar to the advantage game except that the 'deuce' occurs at 30-30. It would seem that this may be a relatively easy type of game for players to adjust to, and so it would appear to be a real alternative to the above two types of games.
4. the 50-40 game. This game has been described by Pollard and Noble (2004). When using this game the server has to reach 50 (1 more point than 40) whilst the receiver has to reach only 40. Thus, under this type of game the server has the advantage of serving but the disadvantage of having to win one more point than the receiver in order to win the game. The 50-40 game has been shown to have greater efficiency than the no ad game when used within a tiebreak set. It also reduces the likelihood of 'long' matches.
5. the 50-40, 40-0, 40-15 game. This game is the same as the 50-40 game described above except that the server also wins the game if the score reaches 40-0 or 40-15.
6. the 50-40, B3 game. This game is the same as the 50-40 game except that, if the score 40-30 is reached (where each player is exactly one point away from winning the game under the 50-40 game rules), the best of 3 points system is used to determine the winner of the game.

In our analyses we have used the parameters $p_A = 0.64$ and $p_B = 0.6$, where p_A is player A's probability of winning a point on service and p_B is player B's probability of winning a point on service. These parameter values are quite representative for men's tennis. To see this, the reader is referred to the paper by Cross and Pollard (2011) which gives values for p_A and p_B for men's grand slam tennis. It should be noted that the observed values for p_A and p_B in this paper by Cross and Pollard are biased ones. The winner's p_A statistic has a positive bias and the loser's p_B statistic has a negative bias (see Pollard, Pollard, Lisle and Cross, 2010).

The systems are applicable to a range of men's singles events including the grand slam events where matches are known to go on for a 'long' time with the advantage final set. By changing the game structure the advantage final set can still be played and the chances of reaching 6 games all are significantly reduced. For example with $p_A = 0.64$ and $p_B=0.60$, the chances of reaching 6 games all for a standard deuce game is 18.9%. Whereas using a 50-40 game the chances of reaching 6 games all are reduced to 12.4%.

Table 1 gives values for the probability player A wins his service game, the expected value and SD of the number of points in such a game, the expected value of the excitement in this game, and the expected value of the excitement conditional on player A winning, and losing. The values in this table and all other tables were determined mathematically or numerically using backwards recursion (Barnett, 2016). They are not the result of simulations. We explain the method by first looking at a single game where we have two players, A and B, and player A has a constant probability p_A of winning a point on serve. We set up a Markov chain model of a game where the state of the game is the current game score in points (thus 40-30 is 3-2). With probability p_A the state changes from a, b to a + 1, b and with probability q_A=1-p_A it changes from a, b to a, b + 1. Thus if $P_A(a,b)$ is the probability that player A wins the game when the score is (a,b), we have:

$$P_A(a,b) = p_A P_A(a+1,b) + q_A P_A(a,b+1)$$

The boundary values are: $P_A(a,b) = 1$ if $a = 4, b \leq 2$, $P_A(a,b) = 0$ if $b = 4, a \leq 2$.

The boundary values and formula can be entered on a simple spreadsheet. The problem of deuce can be handled in two ways. Since deuce is logically equivalent to 30-30, a formula for this can be entered in the deuce cell. This creates a circular cell reference, but the iterative function of Excel can be turned on, and Excel will iterate to a solution. In preference, an explicit formula is obtained by recognizing that the chance of winning from deuce is in the form of a geometric series

$$P_A(3,3) = p_A{}^2 + p_A{}^2\, 2p_Aq_A + p_A{}^2(2p_Aq_A)^2 + p_A{}^2(2p_Aq_A)^3 + \cdots$$

where the first term is $p_A{}^2$ and the common ratio is $2p_Aq_A$.

The sum is given by $p_A{}^2/(1-2p_Aq_A)$ provided that $-1<2p_Aq_A<1$. We know that $0<2p_Aq_A<1$, since $p_A>0$, $q_A>0$ and $1-2p_Aq_A=p_A{}^2+q_A{}^2>0$.

Therefore the probability of winning from deuce is $p_A{}^2/(1-2p_Aq_A)$. Since $p_A+q_A=1$, this can be expressed as:

$$P_A(3,3) = p_A{}^2/(p_A{}^2+q_A{}^2).$$

The rest of the equations follow simultaneously.

Table 1. Some characteristics for a game of player A's service (p_A = 0.64)

	Adv game	**No ad game**	**30-30, adv**	**50-40 game**	**50-40, 40-0, 40-15**	**50-40, B3**
P(A wins a-game)	0.813	0.783	0.787	0.627	0.676	0.649
E(N(a-game))	6.254	5.591	4.873	4.989	4.214	5.485
SD(N(a-game))	2.424	1.049	2.439	0.885	1.092	1.533
E(Ex(a-game))	0.806	0.789	0.777	0.939	0.857	0.990
E(Ex(a-game)/A wins game)	0.713	0.704	0.699	0.918	0.790	0.959
E(Ex(a-game)/A loses game)	1.207	1.098	1.065	0.976	0.976	1.048

Table 2 is a replicate of Table 1 for a game of player B's service.

Table 2. Some characteristics for a game of player B's service ($p_B = 0.60$)

	Adv game	No ad game	30-30 Adv	50-40 game	50-40, 40-0, 40-15	50-40, B3
P(B wins b-game)	0.736	0.710	0.714	0.544	0.600	0.561
E(N(b-game))	6.484	5.697	5.049	4.973	4.273	5.484
SD(N(b-game))	2.590	1.037	2.597	0.920	1.083	1.574
E(Ex(b-game))	1.001	0.929	0.933	0.995	0.929	1.068
E(Ex(b-game)/B wins game)	0.900	0.843	0.852	1.016	0.882	1.084
E(Ex(b-game)/B loses game)	1.284	1.140	1.134	0.971	0.999	1.048

Table 3 gives values for several characteristics of a tiebreak set of tennis. It is assumed that player A serves in the first game of the set. These characteristics include the probability player A wins the set, the expected value and SD of the number of points in the set, the efficiency of the set scoring system, the efficiency relative to a set using advantage games, the expected total excitement in the set, the expected excitement per point played, and the expected excitement per point played relative to a set using the advantage game.

Table 3. Various characteristics for a tiebreak set of tennis using six different game scoring systems when $p_A = 0.64$ and $p_B = 0.60$

	Adv game (1)	No ad game (2)	30-30 Adv (3)	50-40 game (4)	50-40, 40-0, 40-15 (5)	50-40, B3 (6)
P(A wins set)	0.633	0.622	0.622	0.620	0.614	0.628
E(N(set))	64.624	57.001	50.415	48.798	42.112	53.631
SD(N(set))	16.000	13.087	14.068	11.686	10.654	13.163
Efficiency	0.658	0.630	0.713	0.708	0.745	0.734
Rel Eff	1.000	0.957	1.084	1.075	1.132	1.116
E(Ex(set))	2.991	2.770	2.772	2.722	2.603	2.889
SD(N)/E(N)	0.248	0.230	0.279	0.240	0.253	0.245
E(Ex)/E(N)	0.0463	0.0486	0.0550	0.0558	0.0618	0.0539
Rel Excite	1.000	1.050	1.188	1.205	1.33512	1.164

Several observations about a set of tiebreak tennis can be made from Table 3. **Compared to using the advantage game**,

1. the no ad game reduces the probability that player A wins the set by about 0.01, it reduces the average set duration by about 7.6 points, and it reduces the standard deviation of the set duration from 16 points to 13.1 points. The set using the no ad game is about 4.3% less efficient and the excitement per point played is about 5% greater.
2. the 30-30 advantage game reduces the probability that player A wins the set by about 0.01, it reduces the average set duration by about 14.2 points, and it reduces the standard deviation of the set duration from 16 points to 14.1 points. The set using the 30-30 advantage game is about 8% more efficient and the excitement per point played is about 19% larger.
3. the 50-40 game reduces the probability that player A wins the set by about 0.013, it reduces the average set duration by about 15.8 points, and it reduces the standard deviation of the set duration from 16 points to 11.7 points. The set using the 50-40 game is about 7.5% more efficient and the excitement per point played is about 20.5% larger.
4. the 50-40, 40-0, 40-15 game reduces the probability that player A wins the set by about 0.019, it reduces the average set duration by about 22.5 points, and it reduces the standard deviation of the set duration from about 16 points to 10.7 points. The set using the 50-40, 40-0, 40-15 game is about 13% more efficient and the excitement per point played is about 33% larger.
5. the 50-40, B3 game reduces the probability that player A wins the set by about 0.005, it reduces the average set duration by about 11 points, and it reduces the standard deviation of the set duration from about 16 points to 13.2 points. The set using the 50-40, B3 game is about 11.6% more efficient and the excitement per point played is about 16.4% larger.

The ‘shortest’ game that has been approved within the Rules of Tennis is the no ad game. **Compared to using the no ad game**,

1. a tiebreak set of tennis using the **30-30 advantage game** has a similar value for the probability that player A wins the set, it reduces the average set duration by about 6.6 points, but it increases slightly the standard deviation of the set duration from 13.1 points to 14.1 points. The set using the 30-30 advantage game is about 13% more efficient and the excitement per point played is about 13% larger. Overall, it would appear to be an alternative to using the no ad game.
2. a tiebreak set of tennis using the **50-40 game** has a slightly smaller value for the probability that player A wins the set, it reduces the average set duration by about 8.2 points, and it decreases the standard deviation of the set duration from 13.1 points to 11.7 points. The set using the 50-40 is about 12.3% more efficient and the excitement per point played is about 14.8% larger. Overall, it would appear to be an alternative to using the no ad game.
3. a tiebreak set of tennis using the **50-40, 40-0, 40-15 game** has a value for the probability that player A wins the set that is about 0.008 lower, it reduces the average set duration by about 14.9 points, and it decreases the standard deviation of the set duration from 13.1 points to 10.7 points. The set using the 50-40, 40-0, 40-15 game is about 18.2% more efficient and the excitement per point played is about 27.2% larger. Overall, it would appear to be an alternative to using the no ad game.
4. a tiebreak set of tennis using the **50-40, B3 game** has a value for the probability that player A wins the set that is about 0.006 higher, it reduces the average set duration by

about 3.4 points, and it has a similar value for the standard deviation of the set duration. The set using the 50-40, B3 game is about 16.6% more efficient and the excitement per point played is about 10.8% larger. Overall, it would appear to be an alternative to using the no ad game.

Overall, it would appear that each of the four ‘new’ and ‘shorter’ tiebreak set scoring systems studied in this paper could be considered as alternatives to the two present systems approved within the Rules of Tennis.

Further Possible Studies

Some alternatives and variations of the above research that could be studied are now listed.

1. An alternative to the no ad game would be modifying it and playing the best of three points if deuce is reached. This would increase the average duration, but this average duration could be decreased in other ways such as declaring the game over if 40-0 or 40-15 is reached.
2. An alternative to the 30-30 advantage game would be the 30-30, B3 game in which, if 30-30 is reached, the best of three points is played. Note that this is slightly different to the no ad game in that in the 30-30, B3 the score 40-0 is a win to the server (and 0-40 is a loss).
3. Another alternative to the 30-30, B3 game in 2. immediately above is the 30-30, B1 or 30-30, no ad game which is simply the B5 points game. This would obviously lead to a considerably shortened set.
4. It would seem, after looking closely at Table 3, that the 50-40, 40-0, 40-15, B3 game (being a combination of systems (5) and (6) in the study) would be an interesting one to examine (and it is possibly slightly better overall than all of those in Table 3 in terms of the various characteristics being considered). However, some people would believe that it is simply too complicated for practical application.
5. It would seem that if some tournaments are looking for shorter sets, then a simple approach might be to have the first 4 games (in the first set only) the normal advantage games as at present, and replace all remaining games in the match with a shorter version of a game. This hybrid approach would give both players the opportunity to ‘get in their rhythm in the first 4 games’ (a ‘balanced’ number of games in terms of serving and receiving from each end of the court), before things get more important and more exciting.

Conclusions

Four new ‘short’ games of tennis have been considered in this paper.

The first is the 30-30 advantage game. The winner of this game is the first person to win at least 3 points and win at least 2 points more than the opponent. Thus, the server can win 40-0, 40-15, or win after ‘deuce’ is reached at 30-30. Correspondingly, the server can lose 0-40, 15-40, or lose after ‘deuce’ is reached at 30-30. Thus, this game is very similar to the advantage game except that the ‘deuce’ occurs at 30-30.

The second is the 50-40 game. When using this game the server has to reach 50 (1 more point than 40) whilst the receiver has to reach only 40. Thus, under this type of game the server has the advantage of serving but the disadvantage of having to win one more point than the receiver in order to win the game.

The third is the 50-40, 40-0, 40-15 game. This game is the same as the 50-40 game mentioned above except that the server also wins the game if the score reaches 40-0 or 40-15.

The fourth is the 50-40, B3 game. This game is the same as the 50-40 game except that, if the score 40-30 is reached (where each player is exactly one point away from winning the game under the 50-40 game rules), the best of 3 points system is used to determine the winner of the game.

For an advantage game, a no ad game and each of the four ‘short’ games above, various relevant characteristics of a tiebreak set of tennis have been used to compare the systems.

The characteristics were the average number of points played in a set, the probability that the better player wins the set, the efficiency of the set scoring system, the standard deviation of the number of points played in the set, and the average excitement of a point played in the set. A ‘good’ scoring system for a set of tennis was considered to have an appropriate number of points played on average, an appropriate value for the probability that the better player wins, a relatively high efficiency, a relatively high average excitement of a point played, and a relatively small standard deviation of the number of points played.

All four of the new ‘shorter’ game scoring systems listed above were shown to be (overall) improved game scoring systems in terms of the above criteria. It could be argued that the first three of the game scoring systems listed above look the most promising as their reduction in expected duration is greatest.

Some further alternatives and variations on the general theme of this paper are listed in the Further Possible Studies section.

The various tournaments around the world use scoring systems approved by the International Tennis Federation. If a tournament was to use a scoring system not approved by the ITF, it would appear that the results of that tournament would not be used for ranking purposes. The tournament would undoubtedly very quickly fail. If the ITF approves a range of scoring systems, the various tournaments can usefully use this range of scoring systems, and the more relevant and popular ones would succeed.

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