A novel shape transformation approach for quantizing facial expressions

Girija Chetty
Faculty of Information Sciences and Engineering
University of Canberra
girija.chetty@canberra.edu.au

Abstract

In this paper, a novel methodology for facial expression rating is proposed, as the intensity of an emotion evolves from neutral to high expression. The face is modeled as a combination of sectors and their boundaries. An expression change in a face is characterised and quantified through a combination of non-rigid deformations. After elastic interpolation, this yields a geometry-based high-dimensional 2D shape transformation, which is used to register regions defined on query-faces. This shape transformation produces a vector-valued deformation field and is used to define a scalar valued Sector Volumetric Difference (SVD) function, which characterises and quantifies the facial expression. A two-stage expression classification is used with first stage detecting low, medim and high levels of expressions, and the second stage involving a HMM-classifier for recognizing six different facial emotions- anger, disgust, fear, happiness, sadness and surprise. Further, the proposed shape transformation approach is compared with marker based extraction method for extracting facial expression features. The performance evaluation done on a Italian audiovisual emotion database DaFex[1,2], comprising facial expression data from several actors eliciting five different emotions – anger, disgust, fear, happiness, sadness and surprise at different intensities (low, medium and high), shows a significant improvement in expression classification for the proposed shape-transformation approach.

1. Introduction

Facial expression is the main means for human and other species to communicate emotions and intentions [3,25]. Such expressions provide information not only about the affective state, but also about the cognitive activity, temperament, personality and psychopathology of an individual [3, 4, 25].

Facial expression analysis has been increasingly used in basic research on healthy people and in clinical investigations of neuropsychiatric disorders including affective disorders and schizophrenia. Since brain disorders likely affect emotional expressions, and their perception [5, 6, 25], the problem of expression quantification and analysis is an extremely challenging problem due to individual physical facial differences such as wrinkles, skin texture etc. These applications may require efforts to quantify differences in expressiveness, degree of facial mobility, frequency and rate of expression, all of which could be associated with brain disorders. A major neuropsychiatric disorder characterized by deficits in emotional expressiveness is schizophrenia, where "flat affect" is a hallmark of the illness [6, 25]. Similar expression quantification methods find immense applications in human-computer interaction applications, and biometric security systems, where subtle expressions can discriminate emotional state of a person, or discriminate a true client from impostor [7, 25].

Methods of expression rating currently employed are time consuming and depend on subjective judgment of raters. We propose an efficient and automatic expression quantification scheme which is simple, reproducible, robust, objective and fast. In order to develop objective measures of changes in facial expressions, it is necessary to:

(1) quantify the change in expression as the emotion changes in intensity from mild to peak;
(2) quantify the difference in facial expression of the same emotion between normal person and person with a disorder in clinical scenarios or impostor in security system scenarios.
(3) construct a model of an expression using normal person against which the expression hypothesis can be tested.

Quantification of fine-grained structural changes in the face is necessary to capture the subtlety of human expression changes, and this requires advanced,

978-0-7695-3456-5/08 $25.00 © 2008 IEEE
DOI 10.1109/DICTA.2008.96

morphometric tools, [25] . For the proposed approach in this paper, we treat the face as a combination of sectors and their boundaries, for extracting subtle facial expression features. A neutral face is chosen as a template and a face with expression, to be compared to and quantified against the template, is chosen as a query-face. Corresponding regions are identified on each of these faces. We then compute an elastic transformation which warps the template to the query-face, mapping the corresponding boundaries to each other [8, 25]. These regions are chosen such that all the distinctly identifiable features of the face are separated and accounted for (see Fig. 1). Such a shape transformation can be defined for each expression and at varying intensities of expression. The differences can be analyzed on a region wise basis by comparing the corresponding shape transformations.

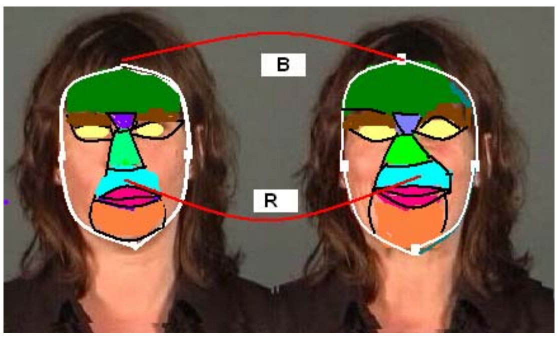

Fig. 1: Regions sectored in the face. B depicts one of the boundaries and R indicates one of the regions. Corresponding regions are painted with the same color. (a) is the neutral face chosen as the template and (b) is the happy face taken as the query-face.

These transformations quantify the volumetric differences among faces with expression. The resultant deformation field characterizes how the neutral face deforms to the face with expression. These deformation fields can be averaged to generate a model for each expression, which can be used for the visual presentation of the quantitative measurements of expression.

2. Facial Expression Quantization Approach

Facial expression quantization technique proposed here involves two stages: expression recognition stage and expression quantification stage. Each of these requires facial modeling. Expression recognition stage was based on HMM-based classifier for classifying the expression as one of the several possible emotions, [10,25]. For expression quantification stage, the intensity of the emotion was quantified on a region-wise basis, to understand how much each region contributes to an expression as well as to its intensity. Automatic expression analysis has attracted attention in computer vision literature because of its importance in clinical investigations, but the efforts have been focused on mainly on expression recognition [10, 11, 12, 13, 14, 15, 16, 25]. We propose a powerful two-stage method involving expression quantification stage and expression classification stage that is applicable region-wise and can be extended to automate the recognition of facial expressions.

Several techniques based on quantitative morphological analysis of complex structures such as the brain and heart, which transform non-rigidly over time, have been developed in medical imaging [17, 18, 19, 25]. These techniques spatially normalize, i.e., elastically register deformable, elastic body parts of a subject to those of a template. We propose to use one such technique called shape transformations to model facial deformations. Shape transformations in general, provide a powerful way to obtain a detailed and localized structural characterization of complex objects such as the brain [20, 21, 8, 25].

In the expression quantification method proposed here, the shape transformation maps a neutral face taken as a template, to a face with expression, which is the query-face. It is based on point correspondences determined on distinct region boundaries along with some landmark points, demarcated on the query-face and template face. We compute an elastic transformation, which warps the template to the query-face, mapping the corresponding boundaries to each other and elastically interpolating the enclosed region [8, 25]. Comparison of the properties of these shape transformations helps to quantify recognized emotions. The advantages of the proposed technique are:

- The method can quantify expression changes between various intensities, e.g. low, medium and high, and across individuals expressing the same or different emotion, by using high-dimensional shape transformations. The current literature on face expression recognition is able to recognize only high emotion, i.e., when all the action units are activated, and not grades of that emotion.

- As the method involves sector based volumetric difference maps using the shape transformations - which quantify the deformation between two faces, the average of each expression is based on these volumetric difference maps. Such models designed using expressions of normal persons allow examining individual differences in people with expression-related disorders. Such analysis, in conjunction with other recognition techniques, can be helpful for comparison of expressions of people with different degrees of neuron-psychiatric disorders. The proposed approach also allows predicting the expression of a neutral face using the deformation field generated by the shape transformation. The shape transformation and the quantification map will be described in detail in Section 3.

We describe the method of computing the shape transformation, and creation of the quantification maps in the next Section. Experiment design for validating the method is described in Section 4. In Section 5, we use the conventional marker based technique used for comparing the proposed approach, and Section 6 discusses the results for different sets of experiments. The paper concludes with Section 7 with some conclusions and further plans.

3. Methodology for quantifying expressions

We choose a neutral face (face with no expression) as a template. All the other expressions of the same individual are categorized as query-faces, to be analyzed against this template. A query-face is one that has expression. The expression could vary in intensity from low to medium to high. Our general approach can be outlined in the following steps:

1. Identify regions on the template that characterize the various facial features. These regions are demarcated by marking their boundaries (as shown in Fig. 1). Some landmark points are also marked; curve segments in between are parameterized via constant-speed parameterization.

2. For each region picked on the template, identify the corresponding region on the query-face.

3. Compute the elastic transformation from one face to the other, so that the sectored regions of the face from the query-face are mapped to their counterparts in the template.

The shape transform provides us with the information regarding the deformation produced by the expression change. Also, we define a sector volumetric differences function (*SVDF*), which provides a numeric value for each pixel on the face. This value quantifies the expansion and contraction of the region on a pixel-wise basis. .

The shape transformation used in this paper is adapted from the work in [8, 25]. Let Ωq and Ωt denote the query space and template space, respectively:

$\Omega t = \{F_t \mid \text{Face with neutral expression}\}$

$\Omega q = \{Ft \mid \text{Face with expression at various intensities}\}\backslash$
(low medium, high)

Now we define each template and query face as a union of regions and their boundaries.

$$F_t = \bigcup_i (B_t^i, R_t^i)$$

$$F_q = \bigcup_i (B_q^i, R_q^i)$$

where B_t^i *and* B_q^i *are the boundaries of the corresponding regions* R_t^i *and* R_q^i *respectively*

Fig. 1 shows some of the regions that have been demarcated on the face. Some landmark points (typically two to four) are also selected on the boundaries. The boundary segments in between two consecutive landmarks are parameterized by a constant-speed parameterization, i.e., by evenly spaced points (after discretization). This parameterization effectively sets up point correspondences between the boundaries of the query-face regions and the boundaries of the corresponding template regions. These point correspondences can be used to define a transformation between the template face and the query-face. In order to determine a shape transformation that accounts for the morphological or elastic changes occurring in the enclosed regions, we compute the elastic transformation S which:

- maps the corresponding points between the boundaries of the template to their counterparts on the query-face boundaries and
- warps the enclosed regions from the template to the query-face, elastically.

The elastic shape transformation *S* is computed as:

$$S : \Omega_t \ni F_t \rightarrow F_q \in \Omega_q$$

- the boundaries are mapped to each other,

$f : B_t^i \rightarrow B_q^i, \forall_i$ by mapping the corresponding points on the boundaries:

And

- the enclosed template regions are elastically warped to the enclosed regions on the subject

$$R_t^i \rightarrow R_q^i$$

Thus S elastically matches the boundaries and accounts for the elastic (deformable) changes not only on the boundary but also within the enclosed region. The resulting shape transformation S quantifies the shape properties of the query-face, *Fq* with respect to the template face, *Ft*. Therefore, two faces with different emotion expression can be analyzed on the basis of a point-wise comparison of the shape transformations.

The shape transformation produces a map of vectors (one vector at each pixel) called a vector field. These vectors provide the direction of movement of the pixel as the result of the deformation caused by the change in expression. Each vector is denoted by a 2-tuple (dx, dy) which denotes the displacement in x and y direction that each pixel on the template undergoes, when it is transformed to the face with expression.

On adding the vector *(dx, dy)* to the position of the template *(x, y)*, we get the deformation *(x + dx, y + dy)* which the pixel on the template undergoes. The position to which the pixel on the neutral template moves to as a result of the action of this vector denotes the deformed position in the query-face after undergoing an expression change. This is known as the deformation field. Additional details of the method can be found in [8, 21, 25]. We obtain two quantities from the shape transformation that we use as expression quantification features for 2nd stage – the expression classification stage:

- The scalar field of values of the sectored volumetric difference function (*SVDF*), which is evaluated at each pixel on the face. We define the SVDF as:

SVDF(s) = det(Γ(S(s))) = determinant of the Jacobian of *S* evaluated at each point s on the query-face.

The *SVDF* value quantifies the volumetric difference (variability in expansion and contraction) between regions. The map containing the *SVDF* values for each pixel of the face is called the *SVD* map of the face. Various inferences may be drawn from the values of the *SVD* function. If

Fig. 2: Information obtained from the shape transformation: (a) template face, (b) query-with expression, (c) intensity normalized SVD map, (d) vector deformation field and (e) color map for visualization of SVD maps.

SVDF1(u) ≥ SVDF2(u) for the same region on two faces which have the same expression, at the same intensity, then it quantifies that the region in face 1 has deformed (expanded or contracted) more than the region in face 2, relative to their respective neutral states. It, therefore, quantifies the variability in expressing the same emotion across individuals. If this is on the same face and same region, it indicates and quantifies the change in expression. Fig. 2(c) shows the color visualization of an SVD map.

- The vector displacement fields of the deformation. These characterize the direction and degree of movement of each pixel of the face during the course of an expression change.

These vectors can be used to quantify temporal changes in an expression. Fig. 2(d) shows the deformation of each pixel of the template face 2(a) as a result of the expression change from 2(a) to 2(b).

For the 2nd stage-the expression classification stage, the two quantification measures - SVD map of the shape transformation of a query-face with respect to a template, fused with the vector deformation field (VDF map), provides information about the direction of movement of the regions, forms the feature sets for training the HMM for each expression classification.

4. Experiment Design

In this section we present some details of the database used and the algorithm implementation issues.

4.1. Database

For all the experiments in the proposed facial expression analysis approach, an audiovisual emotion database DaFEx[1, 2], with actors eliciting six different emotions were used for evaluating the proposed fusion approach. The experiments were not meant to be exhaustive, and the focus is on the applicability of our approach in the investigation of affect processing in persons with expression-related disorders.

DaFEx is an Italian audiovisual database of posed human facial expressions collected with the purpose of creating a valid benchmark, and can also be used as a general reference for research on emotions and facial expressions. DaFEx is composed by 1008 short videos in which Ekman's prototypic emotions (happiness, sadness, anger, fear, disgust and surprise) plus the neutral expression are shown. Facial expressions were recorded by 8 Italian professional actors (4 male and 4 female) on 3 intensity levels (low, medium, high) and in 2 different conditions: The utterance subset comprised the actors playing these emotions while uttering a phonetically rich and visemically balanced sentence ("In quella piccola stanza vuota c_era per_ltanto una sveglia", Italian for: "In that little empty room there was only an alarm clock").

Fig. 3: Some sample images from DaFeX audiovisual emotion corpus [1].

For non-utterance subset, the actors played emotions without pronouncing any sentence. In addition, each video started and ended with the actor showing a neutral expression. Both video and audio signals were recorded. Each actor recorded a sub-set of 126 videos, which includes all the emotions considered, at the three intensity levels and in the two different conditions.

The recording was done with a digital camera (Canon MV630i) and a directional microphone (Sennheiser MKH 406T). Videos were then compressed with Indeo 5.10 compression and audio signal was filtered in order to eliminate external noise. Finally, videos were made available as .avi files with 360 x 288 pixel images. We used the non-utterance subset of this corpus for evaluating the expression quantification approach proposed in this paper. Figure 3 shows some images from this corpus.

4.2. Expression Analysis

In Fig. 2, we show the information that is produced by the shape transformation applied to a segmented face shown in Fig.1. Fig. 2(a) shows the template neutral face and Fig. 2(b) shows the corresponding query-face expressing anger. We then compute the shape transformation that elastically warps the regions demarcated on the template to the corresponding regions identified on the subject (see Fig. 1 for regions). A positive SVD value indicates an expansion and a negative SVD value indicates a contraction. These are the values used in the analyses. However, these SVD values are normalized to a specific range for visualization of the expression changes in the form of a color map. In our case, we choose the range to be 0–90, as it provides the best demarcation. In doing so, the base value of 0, indicating no change, is shifted to 30. The range for displaying the color map can be changed by the user. Fig. 2(c) shows the color coded SVD map of SVD values computed at each pixel of the face, the color map for which is in Fig. 2(e). After normalization, an increase in SVDF values from the template to the subject indicates the expansion and a decrease indicates the contraction of the corresponding region in the subject. These maps are computed at varying intensities of the same emotion, to study expression changes. In general, darker blues indicate contraction and yellow to red depicts increasing expansion. Fig. 2(d) depicts the deformation field indicating the pixel movements due to the expression change from 2(a) to 2(b). In order to create 2(d), we represented the pixels of the template face as a grid of the same size. Then to each position of the grid, the displacement of the vector field produced as a result of the shape transformation, was applied. This produces the deformed grid shown in Fig. 2(d). In this paper, we will use SVD maps (as shown in Fig. 2(c)) for quantification as these provide a numeric value of the changes at each pixel as a result of the expression change.

The SVD maps can also be used to determine movement of facial regions. This is achieved by studying the region as a combination of expansion and contraction of several regions.

In Fig. 2(c), the forehead contracts and the upper lid and the region between the eyes expand. This indicates that the eyebrows have been raised. The mouth and upper lip expand. This in conjunction with the fact that the chin and cheeks expand slightly and lower face expands, indicating a jaw drop. The actual expansion and contraction of these regions and the direction of their movement can be verified with the actual changes in face in 2(a) and (b).

We now describe the conventional marker-based approach for extracting facial expressions for comparison and then describe the details of experiments for the second stage of expression classification based on HMM classifier trained with LDA optimized SVDF and VDF maps.

5. Marker Based Feature Extraction

We considered the high expression dataset for comparing the performance of marker-based approach with shape transformation approach as the expressions from the high-expressions are clearer and easier to distinguish. The local coordinate center of each frame was obtained by translating all the markers in Figure 4, with reference to nose marker. Then the reference frame was picked by choosing the neutral frame with mouth closed, [26]. Each data frame is divided into five blocks: forehead, eye, lower mouth, right cheek and left cheek area (see Figure 4).

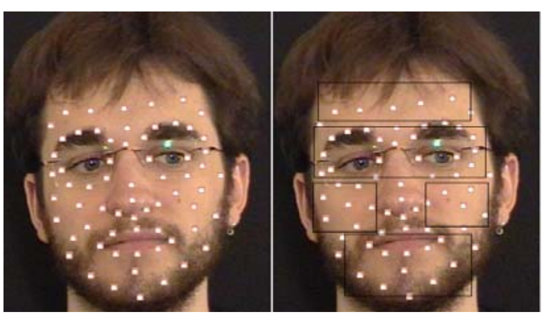

Fig. 4: Visual feature-markers and five face regions for a face images from DaFeX audiovisual emotion corpus [1].

For each block, the *x-y* coordinate of markers in that block were concatenated together to form a data vector. Then, Linear Discriminant Analysis (PCA) method is used to reduce the number of features per frame into a 10-dimensional vector for each area, covering more than 95% of the variation. A 10-dimensional feature vector is obtained for each block. The method user here is similar to a marker based feature extraction technique proposed in [26],

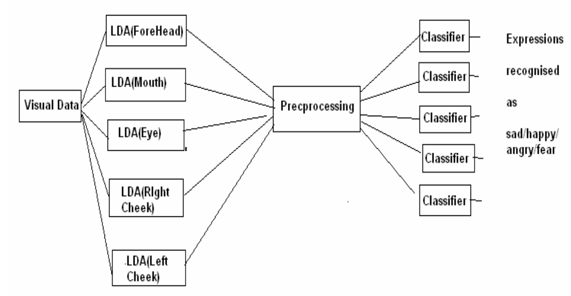

Figure 5: Facial expression feature quantification

6. Results and Discussion

In this section we evaluate the performance of proposed shape transformation approach with the marker based approach for facial expression classification for high level expression data set of the DaFeX database. The shape transformation forms the part of preprocessing stage shown in Figure 5 Figures 6 show the SVD maps for the expressions—happiness, sadness and anger. The SVD maps are color-coded on the basis of the color map shown in Fig. 2(e). The regions identified on the faces have been shown in Fig.1. which depict the pixel-wise SVD values which indicate expansion and contraction. For happy face (first row), the eyes contract, the mouth expands and the cheeks contract indicating a sideways expansion of the mouth. The lower lids expand depicting a cheek raise. The forehead shows no change and neither does the region between the eyes and the eyebrows. The contraction is indicated by a decrease in SVD values from the template to the subject images. The expansion of the regions is indicated by the increase in SVD values from the template to the subjects.

The second row in Figure 6 shows the sadness in varying intensities. The eyes contract, the region between the eyes and the eyebrows expand and the forehead contracts, indicating an eyebrow raise, the upper lip expands, the mouth expands and the cheeks contract. The chin expands and the whole face contracts, indicating a sideways expansion of the lips and the chin. The third row in Figure 6, shows the same analysis carried out for the expression of anger.

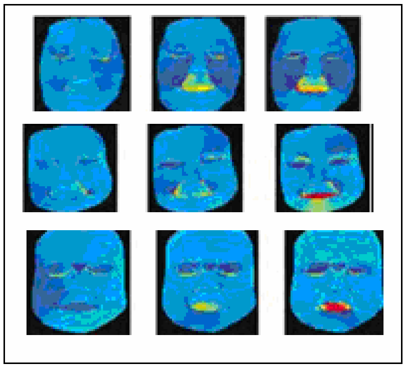

Fig. 6: Quantification of expression for a subject as low, medium and high intensity(First row: happy; Second row: sadness, Thrid row: anger)

The SVD values of pixels in the corresponding regions in the faces for anger, indicate that the eyes contract, the forehead expands and the regions between the eyes contract, indicating a brow lowering. The cheeks contract and the lower lids expand, indicating a cheek raise.

The next set of results presented here, show how the the second stage- the expression classifier performs for the pre-processed quantized expression data for the proposed shape transformation approach. Table 1 shows the recognition performance for the marker based approach as compared to shape transformation approach. As can be observed, from the Table 1, anger, happiness and neutral state are recognized with more than 90 percent of accuracy for the proposed shape transformation approach as compared to marker based approach. For marker based approach, anger is classified with 86.24%, sadness with 80.39%, happiness with 88.38% and neutral with 81.27% accuracy, whereas for shape transformation approach, the performance achieved is 91.35% for anger, 82.66% for sadness, 93.45% for happiness, and 81.66% for neutral expression. For neutral expressions, both marker based and shape transformation approach perform similar. The recognition rate of sadness was in general lower, 80.39% for marker based approach, and 81.66% for shape transformation approach. This emotion is confused with neutral state, because the markers used for five regions on face may not be accurately capturing the classes separately. The happiness performs best for both marker based approach(88.28%) and shape transformation approach(93.45%). The proposed shape transformation performs significantly better for all expressions, but the best for happiness approach. These results suggest that use of shape transformation approach - (SVD+VDF) maps, can explicitly quantify emotions, and allow better recognition of difficult facial characteristics such as subtle expressions.

Table 1: Comparng Recognition performance for different facial expressions : Marker vs. Shape transformation approach.

Expressio n	**Marker based technique**	**Proposed shape Trans. approach**
Anger	86.24 %	91.35%
Sadness	80.39%	81.66%
Happiness	88.38%	93.45%
Neutral	81.27%	81.66%

7. Conclusion and Further Scope

In this paper, we have proposed a simple approach for quantifying the intensity of emotions. The applicability of our approach was demonstrated on various expressions at varying levels of intensity. Further, the technique is able to capture very subtle differences in facial expression change. The results show that the proposed shape transformation features allow better quantification of facial expressions as compared to marker based features. Further work involves a thorough testing of the proposed approach for all the subsets of rich expression data available in DaFEx database.

8. References

1] Battocchi, A.; Pianesi, F.. 2004. DaFEx: *Un Database di Espressioni Facciali Dinamiche*. In Proc. of the SLI-GSCP Workshop "Comunicazione Parlata e Manifestazione delle Emozioni", Padova (Italy) 30 Novembre - 1 Dicembre 2004.

[2]Mana N., Cosi P., Tisato G., Cavicchio F., Magno E. and Pianesi F., *An Italian Database of Emotional Speech and Facial Expressions*, In Proceedings of "5th International Conference on Language, Resources and Evaluation (LREC2006), Genoa, Italy, 24-25-26 May 2006.

[3] Darwin C. *The expression of the emotions in man and animals. London*: John Murray; 1872.

[4] Schmidt KL, Cohn JF. *Human facial expressions as adaptations: Evolutionary questions in facial expression research*. Yearbook Phys Anthropol 2001;44:3–24.

[5] Gainnoti G, Caltagirone C, Zoccolotti P. *Left/right and cortical/subcortical dichotomies in the neuropsychological study of human emotions*. Cognition Emotion 1993;7:71–94.

[6] Rinn WE. *The neuropsychology of facial expression: a review of the neurological and psychological mechanisms for producing facial expressions*. Psychol Bull 1984;95:52–77.

[7]Chetty G., and Wagner M., Liveness verification in audio-visual speaker authentication, Proc.10th Australian International Conference on Speech Science & Technology Australian Speech Science & Technology Association Inc.

[8]Davatzikos C. *Measuring biological shape using geometry-based shape transformations*. Image Vision Comput 2001;19:63–74.

[9] Ekman P, Rosenberg E. *What the face reveals*. Oxford University Press; 1997.

[10] Bartlett MR, Hager JC, Ekman P, Sejnowski TJ. *Measuring facial expressions by computer image analysis.* Psychophysiology 1999;36:253–64..

[11] Black MJ, Yacoob Y. *Recognizing facial expressions in image sequences using local parameterized models of image motion.* Int J Comput Vision 1997;25(1):23–48.

[12] Essa IA, Pentland AP. *Coding, analysis, interpretation and recognition of facial expressions*. IEEE Trans Pattern Anal Mach Intell 1997;19(7):757–63.

[13] Lien J, Kanade T, Cohn J, Li CC. *Detection, tracking and classification of action units in facial expression*. J Robot Autonom Syst 2000;31:131–46.

[14] Terzopoulos D, Waters K. *Analysis of facial images using physical and anatomical models.* In: Proceedings of the International Conference of Computer Vision, 1990.
[15] Yacoob Y, Davis L. *Recognizing human facial expression*. Centre for Automation Research, University of Maryland; 1994.

[16] Zhang Z. *Feature-based facial expression recognition: Sensitivity analysis and experiments with a multilayer perception*. Int J Pattern Recog Artif. Intell 1999;13(6):898–911.

[17] Christensen GE, Joshi SC, Miller MI. *Volumetric transformation of brain anatomy*. IEEE Trans Med Imaging 1997;16(6):864–77.

[18] Cohn JF, Zlochower AJ, Lien J, Kanade T. *Automated face analysis by feature point tracking has high concurrent validity with manual FACS coding.* Psychophysiology 1999;36:35–43..

[18] Collins DL, Evans AC. *Automatic 3D estimation of gross morphometric variability in human brain*. Neuroimage 1996;3(3).

[19] Maintz JBA, Viergever MA. *A survey of medical image registratio*n. Med Image Anal 1998;2(1):1–36.

[20] Bajcsy R, Kovacic S. *Multiresolution elastic matching*. Comput Vision Graphics Image Process 1989;46:1–21.

[21] Davatzikos C, Vaillant S, Resnick JL, Prince S, Levotsky N, Bryan. *A computerized approach for morphological analysis of the corpus callosum*. J Comput Assisted Tomography 1996;20:88–97.

[22] Gur RC, Sara R, Agerdoorn MA, Marom O, Hughett P, Macy L, et al. *A method for obtaining 3-dimensional facial expressions and its standardization for the use in neurocognitive studies*. J Neurosci Meth 2002;115:137–43.

[23] Kohler CG, Turner T, Stolar NM, Bilker WB, Bresinger CM, Gur RE, et al. *Differences in facial expression of four universal emotion*. Psychiat Res 2004. .

[24] Duda RO, Hart PE, Stork DG. Pattern classification. Wiley; 2001.
[25] Verma R., Davatzikosa C., Lougheadb J., Indersmitten T., Ranliang H., Kohlerb C., Gurb R.E., Gurb R.C., *Quantification of facial expressions using high-dimensional shape transformations*, Journal of Neuroscience Methods 141 (2005) 61–73.

[26] Busso C., Deng Z., Yildrim S., Bulut M., M.L. Chu., K Abe., S.Lee., Neuamnn U., Narayanan S., "*Analysis of Emotion Recognition using Facial Expresiions, Speech ad Multimodal Information", in Proceedings ICME 2004 Conference.,*http://graphics.usc.edu/cgit/pdf/papers/ICMI2004-emotionrecog_upload.pdf