# NOISES REMOVAL FOR IMAGES IN NAKAGAMI FADING CHANNELS BY WAVELET-BASED BAYESIAN ESTIMATOR

*Xu .Huang*
Faculty of Information Sciences and Engineering, University of Canberra, ACT 2601, Australia

## ABSTRACT

With the projected significant growth in mobile internet and multimedia services, there is strong demand for wireless image communication. However, a lot of questions related to the multimedia wireless communications raised, such as "how to obtain a good quality images in wireless communication fading channels?" In this paper, we first established Bayesian estimator based on wireless communication modelled by Nakagami fading channel. Our previous research results [4-6] have been extended to this modelling, in particular for multi-noises, such as Gaussian and Poisson noises in Nakagami fading channels. As an example, an improved Bayesian estimator (soft and hard threshold methods), is also illustrated in this paper, not only by traditional assessments but also by "structural similarity" [10].

## 1. INTRODUCTION

The mobile wireless industry is poised for greatness. This industry, over the last few years, has undergone and continues to undergo a tremendous amount of change. The plethora of technology innovations are at the infrastructure and handset level, besides all the applications that are emerging. At the heart of all the technology platforms and handsets introduced is access-being able to access both voice and data services regardless of where the end user is physically located. It is well known that current research on smart and ubiquitous environments can be divided into two major categories. One aims at providing services to the people in the environment by detecting and recognizing their actions. In this paper we are going to focus on the second category, in particular the wireless image transmissions. The Nakagami *m*-distribution has founded many applications in technical sciences. It has been shown by extensive empirical measurement that this distribution is an appropriate model for wireless links [1-3].

A wide variety of fading effect can be modelled as Nakagami fading with different *m* parameters, including Rayleigh and one-side Gaussian fading as special cases when *m* equals to 1 and 0.5, respectively. Nakagami distribution is also suitable for modelling the output statistics of diversity combining system that are employed extensively to mitigate multipath faded effect. It is obvious that generation of correlated Nakagami fading channels is therefore an essential issue for a laboratory test of wireless systems or subsystems to operate in such a fading environment. Some papers have shown it is possible to have flexible algorithm with the ability to generate correlated Nakagami fading branches with arbitrary fading parameters and correlations [2, 3].

Wireless telephones are not only convenient but are also providing flexibility and versatility. According to the nature of a particular application, wireless communications can be used in home-based and industrial systems or in commercial and military environments. One of major proposals of this paper is to present what will happen when an image is communicated by a Nakagami-*m* fading channels. We shall investigate whether our previous methods work for image noises removal via Bayesian estimator [4-6] in the wireless fading channels.

It is well known that noise degrades the performance of any image compression algorithm. For example, emission and transmission tomography images are usually contaminated by quantum noise, which is Poisson noise. Unlike additive Gaussian noise, Poisson noise is signal-dependent, and separating signal from noise is difficult. Several groups have discussed that wavelet sub band coefficients have highly non-Gaussian statistics and the general class of $\alpha$-stable distributions has also been shown to accurately model heavy-tailed noise [8].

Wavelet transform is a powerful tool for recovering signals from noise and has been of considerably interest. In fact, wavelet theory combines many existing concepts into a global framework and hence becomes a powerful tool for several domains of application.

Donoho gives some minimum thresholds for several threshold schemes, titled "universal thresholds" [9]. These explicitly depend on the standard deviation of noise, where the standard deviation is assumed to be known. In practice, the standard deviation can be readily estimated. For some applications the optimal threshold can be computed. An approach different from "universal thresholds" was presented, in which cross-validation is used. Two approaches to cross validation are used, namely ordinary cross validation (OCV) and generalized cross validation (GCV): each is used to minimize the least-squares error between the original (which is the unknown value) function and its estimate based on the noisy observation.

In this paper it is carefully discussed that a wavelet-based for Bayesian estimator that recovers the signal component of the wavelet coefficients in original images from images contaminated by various noises in Nakagami fading channels, which shows that the previous methods [4-6] can be extended to the wireless fading channels.

As an example, a colour image and its image contaminated by various noises will be shown using the discussed method, the assessments which used not only by the traditional method but also by "structural similarity" [10, 11] shows the results we expected.

## *2.* Wave-Based Bayesian Estimator

It is well known that the symmetric alpha-stable distribution ($S\alpha S$) distribution is defined by its characteristic function:

$$\phi(\omega) = \exp(j\delta\omega - \gamma |\omega|^{\alpha}), \tag{1}$$

Here, the parameters $\alpha$, $\gamma$, and $\delta$ describe completely a $S\alpha S$ distribution. The characteristic exponent $\alpha$ controls the heaviness of the tails of the stable density. $\alpha$ can take values in (0,2]; while $\alpha$ = 1 and 2 are the Cauchy and Gaussian cases respectively. There is not closed-form expression known for the general $S\alpha S$ probability density function (PDF). Thus, it is useful when using the principle of maximum likelihood estimation. The dispersion parameter $\gamma$ ($\gamma > 0$) refers to the spread of the PDF. The location parameter $\delta$ is analogous to the mean of the PDF, which, for our following discussion, will be the same assumption as that in [8].

If a variable $\hat{\theta}$ is unbiased it follows that

$$E(\hat{\theta} - \theta) = 0 \tag{2}$$

which can be expressed as:

$$\int_{-\infty}^{\infty} \dots \int (\hat{\theta} - \theta) f_{\vec{\mathbf{x}};\theta}(\vec{\mathbf{x}};\theta) d\vec{\mathbf{x}} = 0 \tag{3}$$

where $\vec{x}(\xi) = [x_1(\xi), x_2(\xi), \dots, x_N(\xi)]^T$ and $f_{\vec{x};\theta}(\vec{x};\theta)$ is the joint density of $\vec{x}(\xi)$, which depends on a fixed but unknown parameter. Following [15-17] we have

$$\mathbf{var}(\hat{\theta}) \geq -\frac{1}{E\{\partial^2 \ln f_{\mathbf{x};\theta}(\mathbf{x};\theta) / \partial\theta^2\}} \tag{4}$$

The function $\ln f_{\vec{x};\theta}(\vec{x};\theta)$ is well known as the "*log likelihood"* function of $\theta$ (LLF). Its maximum likelihood estimate can be obtained from the equation:

$$\frac{\partial \ln f_{\vec{x};\theta}(\vec{x};\theta)}{\partial\theta} = 0 \tag{5}$$

The first order of differential log likelihood function with respect to $\theta$ is called the maximum likelihood (ML) estimate. If the efficient estimate does not exist, then the ML estimate will not achieve the lower bound and hence it is difficult to ascertain how closely the variance of any estimate will approach the bound.

It is noted that the value of about 1.5 is strongly recommended if there is no information about $\alpha$ due to the 2nd order simulations of the LLF for an alpha-stable [6].

## 3. Wireless Fading Channels

If we take the probability density of $\theta$ as $p(\theta)$; and the posterior density function as $f(\theta | x_1, \dots, x_n)$, then the updated probability density function of $\theta$ is as follows:

$$f(\theta | x_1, \dots, x_n) = \frac{f(\theta, x_1, \dots, x_n)}{f(x_1 \dots, x_n)} = \frac{p(\theta) f(x_1, \dots, x_n | \theta)}{\int f(x_1, \dots, x_n | \theta) p(\theta) d\theta} \tag{6}$$

If we estimate the parameters of the prior distributions of the signal $s$ and noise $q$ components of the wavelet coefficients c, we may use the parameters to form the prior PDFs of $P_s(s)$ and $P_q(q)$, hence the input/output relationship can be established by the Bayesian estimator, namely, let input/output of the Bayesian estimator = *BE*, we have:

$$BE = \frac{\int P_q(q) P_s(s) s ds}{\int P_q(q) P_s(s) ds} \tag{7}$$

$P_s(s)$ is the prior PDF of the signal component of the wavelet coefficients of the ultrasound image and $P_q(q)$ is the PDF of the wavelet coefficients corresponding to the noise.

In order to be able to construct the Bayesian processor in equation (7), we must estimate the parameters of the prior distributions of the signal ($s$) and noise ($q$) components of the wavelet coefficients ($d$). Then, we use the parameters to obtain the two prior PDFs $P_q(q)$ and $P_s(s)$ and the nonlinear input-output relationship *BE*.

Consider the moments of the Nakagami distribution NK($m$,Ω). Following [2], we have the probability density function (PDF) of NK($m$,Ω) as below:

$$E[z^r] = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right) \int_0^{\infty} z^{2m+r-1} \exp\left(-\frac{m}{\Omega} z^2\right) dz \tag{8}$$

and we also have

$$E[z^r] = \frac{\Gamma(m + \frac{r}{2})}{\Gamma(m)} \left(\frac{\Omega}{m}\right)^{r/2} \tag{9}$$

The Gamma function Γ(m) is defined by

$$\Gamma(m) = \int_0^{\infty} x^{m-1} e^{-x} dx \tag{10}$$

Our methodology is that following the above processing to obtain a Nakagami source, then to build our BE by equation (7).

We, in the next section, take an example to illustrate the processing for noise removal.

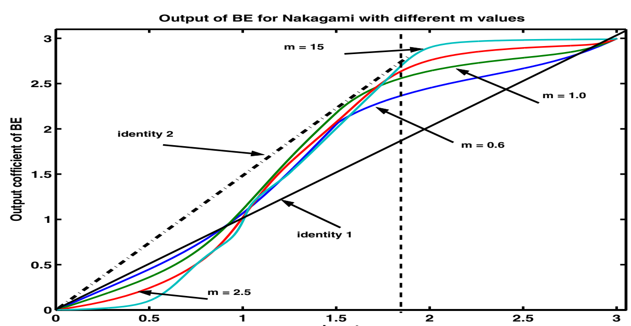


Figure 1: The output of BE for different Nakagami m values, namely $m$ = 0.6, 1.0, 2.5 and 15.

Figure 1 shows our BE in terms of various $m$ values of Nakagami fading channels. We follow the equation (7) and pick four different $m$ values, namely $m$ = 0.6, 1.0, 2.5 and 15. The corresponding Nakagami PDFs are shown in Figure 2.

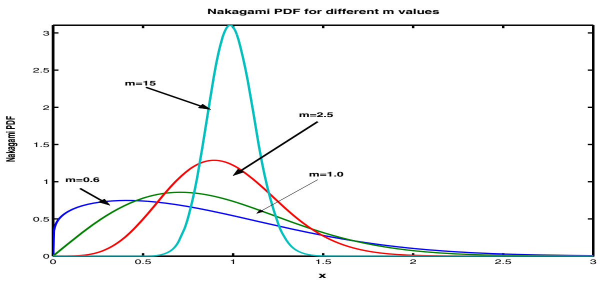


Figure 2: The PDFs of Nakagami fading channels for different $m$ values shown in Figure 2.

From Figure 1 we can see that the different $m$ values make the output of BE are different. The identity line in Figure 1 is labelled and it is well known that the smaller value is the more fading effects would be for the Nakagami fading channels. If we take the identity line in Figure 1, we may observe that the curve makes the output of the BE "soft threshold" like and the cure corresponding to $m$ = 0.6 closer to the "hard threshold". In fact the curves of the output of the BE based on Nakagami with different m values for the fading channels are different, in terms of shapes, from that in our previous results [4-7], which is inspected. But the tendencies of those curves of the BE for the different $m$ values are similar to those previous results in comparison of that in [4-7].

From Figure 2, we can easily find that the bigger values of Nakagami $m$ will be corresponding to the more symmetric of the PDFs curves of the Nakagami fading channels.

Therefore, we can put the Nakagami fading channels for a fixed $m$ together with Poisson and Gaussian noise for an established BE (we call a bland noise removal due to we did not know the detail information about noise) as shown in Figure 3.

In Figure 3, we have used the $\alpha$ = 1.5 that is the results obtained in [6] and the g/mean = 5, 10, 20, 30, and 35 for fixed Poisson distribution with the mean = 10. This is because of the Poisson mean number goes more than 10 we can easily find that the distribution shape close to symmetric.

As we discussed in the section I that we consider the Poisson noise is due to the photo noise due to the X-ray image and the communication channel will contaminated by the Gaussian noise and the wireless noise would contribute by the Nakagami fading.

When BE has been established we can used the wavelet to encode the image to be denoised and then in the wavelet region we use the BE to filter the noise then we decode the wavelet and back to the removal image, which is shown by the following section.

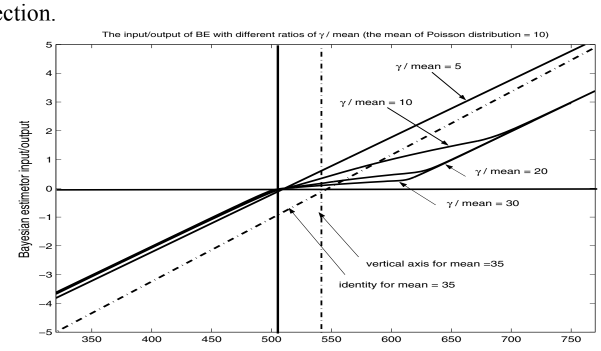


Figure 3: The input/output of BE with different ratios of g/mean with the fixed mean of Poisson mean 10, $\alpha$ = 1.5)

## 4. A Case Study

We first to have a Nakagami fading channels resources built by the methodology introduced in [2] (or [3]). Hence, it is expected to generate an $n$-by-1 correlated Nakagami vector $z$ with fading parameter m and covariance matrix $\boldsymbol{R}_z$.


Figure 4: An original image of the "Dust" in NSW Australia on 12th November 2002.

In order to test the discussed method, we put various noises in the shown image and it is noted that the contaminated image will be also suffering the noise from the fading wireless channel as shown in Figure4.

When we make measurements, we have no information about the noise value of the image we obtained. The only information one may have is from experience in judgment of the noise level, which becomes the outline of the denoising strategy.

In our experimental design, we have sent an image via the wireless communication via WiFi systems (we called UCWiFi, with the speed 54.0 Mbps) set in our wireless laboratory and receiver with a contaminated wireless channel (including Gaussian noise and Poisson noise and Nakagami noise) obtained the received image. The Nakagami fading channel contaminate the image shown in Figure 4, where the $m$ deliberately chose 2.5 we supposed that the BE did not know this parameter (as we shown that the "denoise" will not use this parameter). In fact the multi-noise in real life will be not supposed to know before the noise removal processing.


Figure 5: An Image of the "Dust" in NSW Australia shown in Figure 4 is contaminated by Poisson and Gaussian and Nakagami fading channel.


Figure 6: The result of for the "matched BE" for the Figure 4


Figure 7: The result of for the "matched BE" and known *m* value for the Figure 4

As we previously described that in our papers [4-7] we used the so-called blind "noise removal" to denoise the contaminated image by Nakagami fading channel via the designed BE. The result of the blind noise removal is shown in Figure 6.

If we know some information about the natures of Nakagami fading channel, which is possible even we have some experience in some situations. Also it is noted that we can estimate the *m* value by some cases we are sitting on better position to denoise.

Then we can apply an almost matched BE to the contaminated image by the Nakagami fading channel. For this example, we if we know the matched BE with the $m$ = 2.5 then the denoised result is shown in Figure 7, which is better results than that shown in Figure 6. We put the all the results in Table 1, where we used the signal over mean square error (S/MSE) to measure the "quality" of noise removal of the image.

TABLE 1: THE RESULTS FOR THE NOISE REMOVAL OF THE CONTAMINATED BY THE POISSON, GAUSSIAN, AND NAKAGAMI FADING CHANNEL VIA "BLIND NOISE REMOVAL" AND "MATCHED NOISE REMOVAL". HERE COMPARISON OF DENOISING RESULTS WITH BE ARE IN SIGNAL TO MEAN SQUARE ERROR (S/MSE) IN dB. HERE "METHODS" ARE DEFINED AS 1= SOFT THRESHOLDING; 2 = HARD THRESHOLDING; 3 = HOMOMORPHIC WIENER; 4 = BE (MEAN =10), 5 = BE (POISSON MEAN =10, □ = 1.5 AND □/MEAN =35, AND M =2.5).

| Method | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| S/SME | 14.21 | 14.11 | 13.51 | 14.87 | 15.68 |
| Q | 0.67 | 0.5911 | 0.512 | 0.911 | 0.935 |

In order to make a comparison, a noise removal by traditional hard threshold is shown in Figure 8.


Figure 8: The noise removal of the Figure 6 by hard thresholding.

As the definition of "quality index", Q, [10, 11] is calculated for the individual method listed in the table 1.

## 5. Conclusion

With the growth of mobile communication services and uses, wireless multimedia communications have received considerable attention by both academia and industry in recent years, motivated by the interesting and appealing applications powered up by wireless technology advances. Advances in wireless technology and portable computing along with demands for greater user mobility have provided a major impetus toward development of an emerging class of self-organizing, rapidly developable network architectures for example wireless telmedical systems. The high quality, stable wireless transitions are extremely needed.

It is also well known that the so-called "last mile communication" has drawn great interests and the question obviously raised that how to obtain a good quality images in the decaying and noisy wireless communication channels.

We have been involved in noise removal and wanted to investigate if the technique using the wavelet-based Bayesian estimator can be extended to the wireless communication channels.

The Nakagami fading channels was used with different *m* parameters. The statistician's Bayesian estimator theory is used not only to simplify the selection of parameters but also in some situations to provide more precise images than other methods.

It is noted that if we use the so-called matched BE with an estimation of *m* parameter, the result would be most encouraging as shown in this paper.

## REFERENCES


[1] M. Nakagami, "The m-distribution—A general formula of intensity distribution of rapid fading," Statistical Methods in Radio Wave Propagation, 1960.

[2] Zhefeng Song, Keli Zhang, and Yong Liang Guan, "Generating Coorelated Nakagami Fading Signals with Arbitrary Correlation and Fading Parameters," pp1363-1367, IEEE 2002.

[3] Q. T. Zhang, "A Decomposition Technique for Efficient Generation of Correlated Nakagami Fading Channels," IEEE Journal on selected areas in commun. vol. 18, no.11 pp2385-2392. Nov. 2000.

[4] X. Huang, A.C. Madoc, and A.D. Cheetham, "Image Multi-Noise Removal by Wavelet-based Bayesian Estimator", 2005 IEEE International Symposium on Circuits and Systems, Kobe, Japan, May 23-26, 2005. Accepted (informed on 29/12/04).

[5] X. Huang, A.C. Madoc, and A.D. Cheetham, "Multi-Noise Removal from Images by Wavelet-based Bayesian Estimator", IEEE Sixth International Symposium on Multimedia Software, Miami, FL, USA, December 13-15, 2004. pp.258-264.

[6] X. Huang, A. C. Madoc, and A.D.Cheetham, "Wavelet-based Bayesian estimator for Poisson noise removal from images", IEEE International Conference on Multimedia and Expo, Maryland, U.S. July 6-10, 2003. Vol.I, pp593.

[7] Gamhewage C. de Silva, Toshihiko Yamasaqki, and Kiyoharu Aizawa, "An Interactive Multimedia Diary for the Home," IEEE Computer Society, May 2007, pp52-59.

[8] A. Ben Hamza and Krim, "Image Denoising: A Nonlinear Robust Statistical Approach", IEEE Tran. On Signal Proc. Vol.49, no.12, pp3045-3054, 2001.

[9] D.L. Donoho and I.M. Johnstone, "Wavelet shrinkage: Asymptopia?", Journal of the Royal Statistical Society, Series B, vol. 57, 301-369, 1995.

[10] Zhou Wang, Alan Conrad Bovik Hamid Rahim Sheikh, and Ero P. Simoncelli, "Image Quality Assessment: From Error Visibility to Structural Similarity," IEEE Transactions on Image Processing, Vol. 13, No.4, pp. 600-612, April 2004.

[11] Zhou Wang, Alan C. Bovik and Ligang Lu, "Why is Image Quality Assessment so Difficult?", IEEE International Conference on Acoustics, Speech, & Signal Processing, May 2002.