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Huang, X., & Sharma, D. (2006). Analysis of the Error Probability of a Binary Receiver in Wireless Fading Channels. In D. H-H. Lee (Ed.), *The 8th International Conference on Advanced Communication Technology. Toward the Era of Ubiquitous Networks and Societies* (pp. 1087-1090). Republic of Korea: IEEE, Institute of Electrical and Electronics Engineers. <https://doi.org/10.1109/ICACT.2006.206161>

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Analysis of the Error Probability of a Binary Receiver in Wireless Fading Channels

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Abstract — Mobile and wireless communications are rapidly developing, “what lies ahead for cellular technology?” published by George Lawton [1] shows this concern. Binary receiver in wireless fading channels is one of important issues in the digital wireless communications. The error probability of binary in Nakagami- m fading channels was investigated in this paper. In particular, the application for optimum binary receiver was carefully presented for the so called optimum threshold for an error probability. At the same time, the related Nakagami- m parameter fading channels were also estimated by closed forms, which can be applied to the based estimation for the digital communication systems with various noises. As one application, the error probability was used to estimate the bottom line of binary communication systems. Other digital coding systems will be discussed in separated papers.

Keywords — mobile computing, digital communications, wireless communications, Nakagami m - fading channels, symbol error rate.

1. Introduction

In digital communication systems, the symbol-error rate (SER) has been used very extensively as a performance measure, and accurate methods for evaluating it over fading channels has been an area of interest [2-5]. It is well know that the received signal at a mobile radio usually suffers from fading due to multipath propagation. Different models have been used in the literature to characterize the fading envelope of a received signal. The Nakagami m -fading distribution fading model [6] is one of the most versatile, in the sense that it has greater flexibility and accuracy in matching some experimental data than Rayleigh, log-normal or Rician distributions [7-9].

Annamalai and Tellambura discussed the error probability of binary and M -ary signals in Nakagami fading channel [10], where it based on the closed form solutions to the average SER for a broad class of binary and M -ary modulation formats in Nakagami fading with positive integer m , using some trigonometric identities and the moment generating function (MGF) based analysis method. Recently it was reported that a new closed form formulas for the exact average symbol-error rate of binary and M -ary signal over Nakagami m -fading channels with arbitrary fading index m were obtained [5].

In this paper we are going to focus on how to apply the binary-error rate (M -ary will be discussed later) to the communication systems modeled by Nakagami- m fading channels, in particular for the so called optimum threshold.

2. The optimum threshold detection

Over a certain binary channel, message $x = 0$ and 1 are transmitted with equal probability using a positive and a negative pulse, respectively. The received pulse corresponding to 1 is $p(t)$ and the received pulse corresponding to 0 is $-p(t)$. Assume the peak amplitude of $p(t)$ is A_p at $t = T_p$ as shown in Figure 1. Because of the channel noise $n(t)$, the received pulses will be $\pm p(t) + n(t)$. To detect the pulses at the receiver, each pulse is sampled at its peak amplitude. In the absence of noise, the sampler output is either A_p (for $x = 1$) or $-A_p$ (for $x = 0$). Because of the channel noise, the sampler output is $\pm A_p + n$, where n is the noise amplitude at the sampling instant.

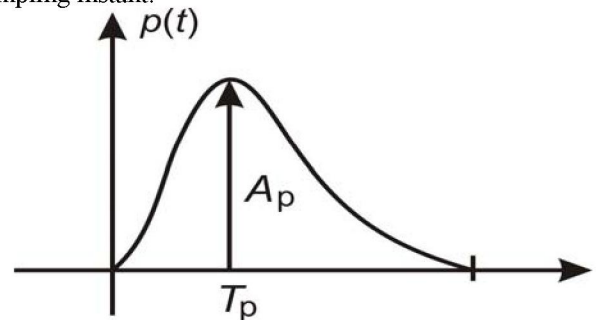


Figure 1: Positive pulse

For example, Gaussian noise would be:

$$p_n(n) = \frac{1}{\sigma_n \sqrt{2\pi}} e^{-n^2 / 2\sigma_n^2} \quad (1)$$

It is important to note that $A_p \in [-\infty, \infty]$, and the sample value $-A_p + n$, for example, can be occasionally be positive, causing the received 0 to be read as 1 as shown in Figure 2, the point (c), where error occurs. Let $P(e|0)$ is the error probability

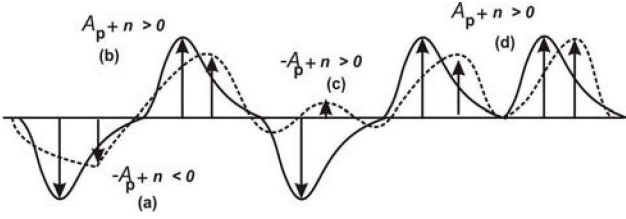


Figure 2: Error probability in threshold detection

given that $\mathbf{0}$ is transmitted, then we have: $P(e|\mathbf{0}) = P(n > A_p)$. Then we can obtain:

$$P(e|\mathbf{0}) = Q(A_p/\sigma_n) \quad (2)$$

where $Q(x)$ is the area under $p_n(n)$ from x to ∞ . Similarly, we have

$$P(e|\mathbf{1}) = P(n < -A_p) = Q(A_p/\sigma_n) = P(e|\mathbf{0}) \quad (3)$$

Let $p(t)$ and $q(t)$ be the two pulses used to transmit $\mathbf{1}$ and $\mathbf{0}$. The optimum receiver structure considered here is shown in Figure 3. The incoming pulse is transmitted through a filter $H(\omega)$, and the output $r(t)$ is sampled at T_b , where T_b is transmitted time period. The decision as to whether 0 or 1

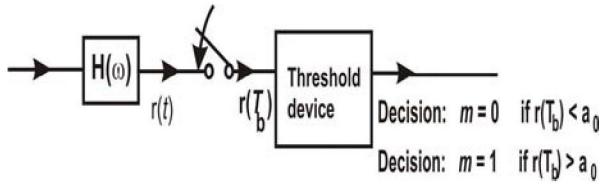


Figure 3: Optimum binary threshold detection

was present at the input depends on where $r(T_b) <$ or $> a_0$, where a_0 is the optimum threshold. To minimize P_e , the error probability, we need to maximize $\rho = A_p/\sigma_n$, because $Q(\rho)$ decreases monotonically with ρ , which is the signal amplitude to rms noise ratio.

3. An Error probability of binary

Let $p_0(t)$ and $q_0(t)$ be the response of $H(\omega)$ to inputs $p(t)$ and $q(t)$, respectively. We have

$$p_0(T_b) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\omega) H(\omega) e^{j\omega T_b} d\omega \quad (4)$$

$$q_0(T_b) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Q(\omega) H(\omega) e^{j\omega T_b} d\omega \quad (5)$$

and σ_n^2 , the variance, or power, of the noise at the filter output, is:

$$\sigma_n^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_n(\omega) |H(\omega)|^2 d\omega \quad (6)$$

If n is the noise output at T_b , then the sampler output $r(T_b) = q_0(T_b) + n$ or $p_0(T_b) + n$, depending on whether $x = \mathbf{0}$ or $x = \mathbf{1}$, is received. The conditional error probability $P(e|x=\mathbf{0})$ is the probability of making a wrong decision when $x = \mathbf{0}$. This is simply the area A_0 under $p_{r|x}(r|\mathbf{0})$ from a_0 to ∞ . Similarly, $P(e|x=\mathbf{1})$ is the area A_1 under $p_{r|x}(r|\mathbf{1})$ from $-\infty$ to a_0 . We have:

$$P = \sum_x P(e|x)P(x) = (A_0 + A_1)/2 \quad (7)$$

For the symmetric case, we have the sum $(A_0 + A_1)$ of the areas is minimized by choosing a_0 at the intersection of the two PDFs, which is $a_0 = [p_0(T_b) + q_0(T_b)] / 2$. Therefore, the corresponding P_e becomes:

$$\begin{aligned} P_e &= P(e|\mathbf{0}) = P(e|\mathbf{1}) \\ &= \frac{1}{\sigma_n \sqrt{2\pi}} \int e^{-[r - q_0(T_b)]^2 / 2\sigma_n^2} dr = Q\left[\frac{a_0 - q_0(T_b)}{\sigma_n}\right] \\ &= Q\left[\frac{p_0(T_b) - q_0(T_b)}{2\sigma_n}\right] = Q\left[\frac{\beta}{2}\right] \end{aligned} \quad (8)$$

where

$$\beta = \frac{p_0(T_b) - q_0(T_b)}{\sigma_n} \quad (9)$$

Submitting equations (4-6) into equation (9), we have

$$\beta_{\max}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|P(\omega) - Q(\omega)|^2}{S_n(\omega)} d\omega \quad (10)$$

hence, the optimum filter $H(\omega)$ is given by [10]:

$$H(\omega) = k \frac{[P(-\omega) - Q(-\omega)]e^{-j\omega T_b}}{S_n(\omega)} \quad (11)$$

where k is an arbitrary constant. So if we choose white noise $S_n(\omega) = \mathcal{N}/2$, we have $\beta_{\max}^2 = (E_p + E_q - 2E_{pq})/2$, where E_p and E_q are the energies of $p(t)$ and $q(t)$, respectively and

$$E_{pq} = \int_0^{T_b} p(t)q(t)dt \quad (12)$$

Therefore, we have the bit error probability or bit error rate (BER) as

$$P_e = Q\left(\frac{\beta_{\max}}{2}\right) \quad (13)$$

for white noise, from above we have:

$$P_e = Q\left(\sqrt{\frac{E_p + E_q - 2E_{pq}}{2N}}\right) \quad (14)$$

4. Error probability of binary in Nakagami fading channels with its estimation of m -parameter

Follow [5], we let the transmitted signal is received over slowly varying flat fading channels, and the γ denote the instantaneous signal to noise ratio (SNR) defined as $\gamma = x^2 E_s/N_0$, where x is the fading amplitude, E_s is the energy per

symbol, and N_0 is the one-side noise spectral density. For Nakagami fading, the probability density function (PDF) of x is given by:

$$p_\gamma(\gamma) = \frac{1}{\Gamma(m)} \left(\frac{m}{\gamma}\right)^m \gamma^{m-1} e^{-m\gamma/\langle\gamma\rangle}, \quad \gamma \geq 0 \quad (15)$$

where, $\Gamma(\cdot)$ is the gamma function, $\Omega = E[x^2]$ denotes the mean square value, and the m is the fading severity parameter and $m \in (0.5, \infty]$. The PDF and the moment generating function (MGF) are given by [4,12]:

$$p_x(x) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{2m-1} e^{-mx^2/\Omega}, \quad x \geq 0 \quad (16)$$

$$\phi_\gamma(s) = \int_0^\infty e^{-s\gamma} p_\gamma(\gamma) d\gamma = \left(1 + \frac{s\langle\gamma\rangle}{m}\right)^{-m}, \quad m \geq \frac{1}{2} \quad (17)$$

where $\langle\gamma\rangle = \Omega E_s / N_0$ denotes the average SNR per symbol. From [4] we have the average bit error rate for coherent binary signals is given by:

$$P_e = \frac{1}{\pi} \int_0^{\pi/2} \phi_\gamma\left(\frac{g}{\sin^2 \theta}\right) d\theta \quad (18)$$

where $g=1$ for coherent binary phase shift keying (BPSK), $g=0.5$ for coherent orthogonal binary frequency shift keying (OBFSK), and $g=0.715$ for coherent CBFSK with minimum correlation. By change of the variable $t = \cos^2 \theta$, after manipulations, equation (18) can be expressed in closed form [4] as below:

$$P_e = \frac{\phi_\gamma(g)}{2\pi} \int_0^1 t^{0.5} (1-t)^{m-0.5} \left(1 - \frac{t}{1 + \frac{g\langle\gamma\rangle}{m}}\right)^{-m} dt \quad (19)$$

$$= \frac{\phi_\gamma(g)\Gamma(m+\frac{1}{2})}{2\sqrt{\pi}\Gamma(m+1)} {}_2F_1\left(m, \frac{1}{2}; m+1; \frac{1}{1 + \frac{g\langle\gamma\rangle}{m}}\right)$$

where ${}_2F_1(a;b;c;z)$ is the Gauss hypergeometric function [13]. Now combining equations (13) and (19), we have

$$Q\left(\frac{\beta_{\max}}{2}\right) = \frac{\phi_\gamma(g)\Gamma(m+\frac{1}{2})}{2\sqrt{\pi}\Gamma(m+1)} {}_2F_1\left(m, \frac{1}{2}; m+1; \frac{1}{1 + \frac{g\langle\gamma\rangle}{m}}\right) \quad (20)$$

This means that we would keep the binary error in balance for the Gaussian white noise and Nakagami noise. In this situation, we can solve the bottom line case for the particular m parameter for the binary error probability. It is well known that $Q(x)$ is widely tabulated and can be found in most of the standard mathematical tables. It can be shown as [11]:

$$Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} \quad \text{for } x \gg 1 \quad (21)$$

Therefore, we can utilize equations (21), (10), and (20) to obtain the nature of Nakagami fading channels and its estimation of m -parameter via the Error probability.

5. Example for white noise

Now we are going to use the white noise as an example for equation (13). The optimum threshold a_0 is obtained by substituting equations (4), (5), and (11) into the equation: $a_0 = [p_0(T_b) + q_0(T_b)]/2$ as shown above and we have

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} P(\omega) Q(-\omega) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(-\omega) Q(\omega) d\omega = E_{pq} \quad (22)$$

This gives:

$$a_0 = (E_p - E_q)/2$$

Next we are going to take two cases, (1) taking the $m = 0.5$ for Nakagami noise for binary situation; (2) taking $m = 2.5$ for comparison to the case (1). It is well known that we have an integral format [14-15] for the Gauss hypergeometric function as below:

$${}_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1} (1-t)^{c-b-1} (1-tz)^{-a} dt \quad (23)$$

Submitting the $m = 0.5$ and 2.5 to about equation for two cases, we can obtain the results from equation (20). In this example we fixed the left hand side of equation (20) as Gaussian white noise and two cases, namely $m = 0.5$ and $m = 2.5$ are in right hand side (RHS) of equation (20). The ratio of two sides against the m parameter values are shown in Figure 4. In order to show the equivalent Gauss hypergeometric function for both two cases, we put them in Figure 5.

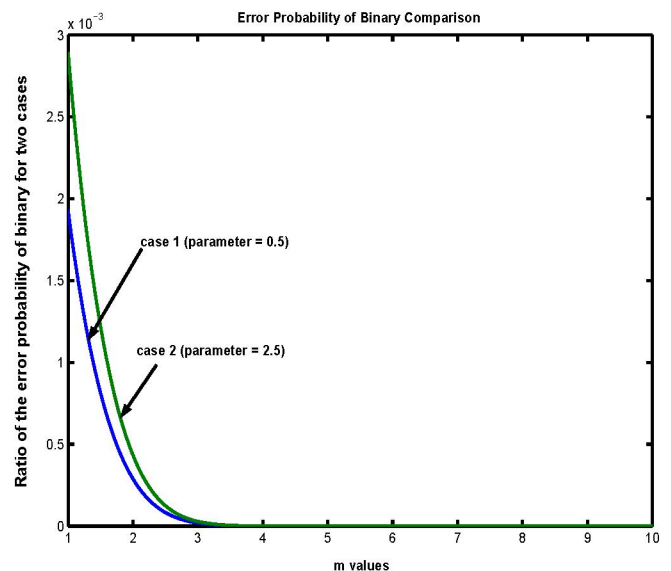


Figure 4: Comparison of error probability of binary in Nakagami Fading Channel.

From Figure 4, we can see that the m smaller the ratio bigger as we expected. This is because the RHS is white noise. It is noted that the curve for parameter being 0.5 is bigger than that being 2.5 for the ratio for the same m value. This is because the Nakagami parameter natures. Figure shows the differences between the equivalent Gauss hyper-geometric function for the case demonstrated in Figure 4, which presented the evidences that when we establish a model for wireless, such as mobile communications, we do need the Nakagami model.

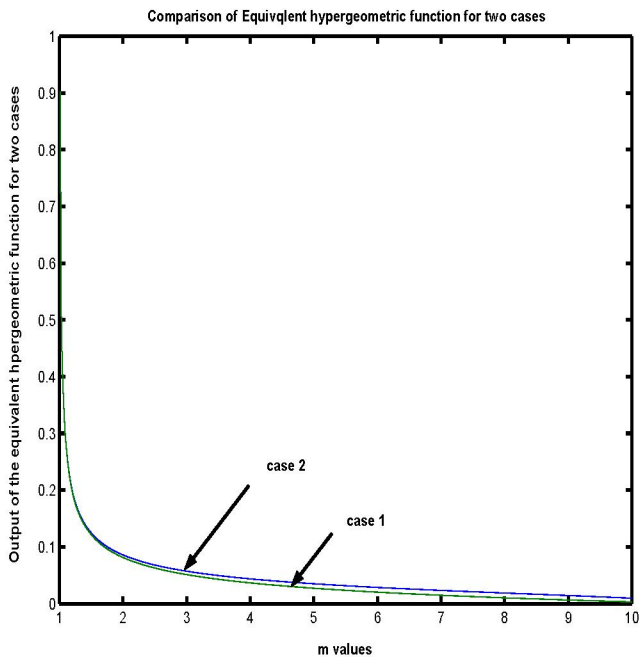


Figure5: Comparison of the equivalent Gauss hyper-geometric function for both two cases.

6. Conclusion

The cellular or mobile telephone is the modern equivalent of Marconi's wireless telegraph, offering two-part, two-way communication. The current generation of wireless devices is built using digital technology. Digital networks carry much more traffic and provide better reception and security than analog networks. The binary receiver in Nakagami fading channels is one of critic issues in the digital wireless communications. We have discussed the error probability of binary in Nakagami fading channel and optimum threshold for binary filters. We established evaluation method by which we have the conclusion that the white noise would be seriously affect the communication channels modeled by Nakagami fading channels. It is interesting to note that we can extend the application to estimation of running parameters for the Nakagami fading channels. This would be useful for designing wireless communication systems.

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