

IMAGE MULTI-NOISE REMOVAL BY WAVELET-BASED BAYESIAN ESTIMATOR

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ABSTRACT

Images are in many cases degraded even before they are encoded. The major noise sources, in terms of distributions, are Gaussian noise, Poisson noise and impulse noise. Noise acquired by images during transmission would be Gaussian in distribution, while images such as emission and transmission tomography images, X-ray films, and photographs taken by satellites are usually contaminated by quantum noise, which is Poisson distributed. Poisson shot noise is a natural generalization of a compound Poisson process when the summands are stochastic processes starting at the points of the underlying Poisson process. Unlike additive Gaussian noise, Poisson noise is signal-dependent and consequently separating signal from noise is more difficult. In our previous papers we discussed a wavelet-based maximum likelihood for Bayesian estimator that recovers the signal component of wavelet coefficients in original images using an alpha-stable signal prior distribution. In this paper, it is demonstrated that the method can be extended to multi-noise sources comprising Gaussian, Poisson, and impulse noises. Results of varying the parameters of the Bayesian estimators of the model are presented after an investigation of α -stable simulations for a maximum likelihood estimator. As an example, a colour image is processed and presented to illustrate the effectiveness of this method.

1. INTRODUCTION

It is well known that noise degrades the performance of any image compression algorithm. In many cases the image is degraded even before it is encoded. Linear filtering techniques have been used in many image-processing applications. They are attractive due to their mathematical simplicity and efficiency in the presence of pure additive Gaussian noise. However, they also blur sharp edges, make some distortions of lines and fine image details, less effectively remove tailed noise, and poorly treat the presence of signal-dependent noise. For example, emission and transmission tomography images are usually contaminated by quantum noise, which is Poisson in nature. Unlike additive Gaussian noise, Poisson noise is signal-dependent, and separating signal from noise is a difficult task. Several groups have discussed that wavelet subband coefficients have highly non-Gaussian statistics [2-7] and the general class of α -stable distributions has also been shown to accurately model heavy-tailed noise [5-7]. It would be very interesting to investigate if α -stable distributions can be still used to the case of signal-dependent noise, such as signals contaminated by Poisson noise and the even more complex cases such as images contaminated by both Poisson and Gaussian noise.

Wavelet transforms as a powerful tool for recovering signals from noise has been of considerably interest [5, 9-12]. In fact, wavelet theory combines many existing concepts into a global framework and hence becomes a powerful tool for several domains of application.

As mentioned by Achim *et al.* [12], there are two major drawbacks for thresholding. One is that choice of the threshold is always *ad hoc*; another is that the specific distributions of the signal and noise may not be well matched at different scales.

These explicitly depend on the standard deviation of noise, where the standard deviation is assumed to be known. In practice, the standard deviation can be readily estimated using the methods discussed in [9], [13]. For some applications the optimal threshold can be computed. An approach different from "universal thresholds" is presented by Nason [14], in which cross-validation is used. Two approaches are used, namely ordinary cross validation (OCV) and generalised cross validation (GCV): each is used to minimize the least squares error between the original (which is the unknown value) function and its estimate based on the noisy observation.

Modelling the statistics of natural images is a challenging task due to the high dimensionality of the signal and the complexity of statistical structures that are prevalent in such images. Numerous papers that discuss modelling the statistics of natural images, including Bayesian processing, presuppose proper modelling of the prior probability density function of the signal. These papers deal with Gaussian noise, or with symmetric stochastic distributions [5, 6, 15,16,17,19,20].

In this paper, our previous discussion [1] is extended to a wavelet-based maximum likelihood for Bayesian estimator that recovers the signal component of the wavelet coefficients in original images from images contaminated by Poisson and Gaussian noise using an alpha-stable signal prior distribution.

As an example, an original colour image and a copy contaminated by Poisson, Gaussian and Impulse noise is used to demonstrate the technique. It is important to note that knowledge of the noise parameters is not a requirement for application of the method; the Bayesian estimator did not know the noise parameters. The parameters shown in the figures are only there to allow us to assess the effectiveness of the technique. The final result shows that the method works well.

2. ALPHA-STABLE DISTRIBUTIONS AND LOG LIKELIHOOD

It is well known that the symmetric alpha-stable distribution ($S\alpha S$) is defined by its characteristic function:

$$\phi(\omega) = \exp(j\delta\omega - \gamma |\omega|^\alpha), \quad (1)$$

The parameters α , γ , and δ describe completely a $S\alpha S$ distribution. The characteristic exponent α controls the heaviness of the tails of the stable density. α can take values in (0,2]; while $\alpha = 1$ and 2 are the Cauchy and Gaussian cases respectively. There is no closed-form expression known for the general $S\alpha S$ probability density function (PDF). Thus, it is helpful when using the principle of maximum likelihood estimation. The dispersion parameter γ ($\gamma > 0$) refers to the spread of the PDF. The location

parameter $\hat{\delta}$ is analogous to the mean of the PDF, which, for our following discussion, will be the same assumption as that in [5].

If a variable $\hat{\theta}$ is unbiased it follows that

$$E(\hat{\theta} - \theta) = 0 \quad (2)$$

which can be expressed as:

$$\int_{-\infty}^{\infty} \dots \int (\hat{\theta} - \theta) f_{\bar{\mathbf{x}};\theta}(\bar{\mathbf{x}};\theta) d\bar{\mathbf{x}} = 0 \quad (3)$$

where $\bar{\mathbf{x}}(\xi) = [x_1(\xi), x_2(\xi), \dots, x_N(\xi)]^T$ and $f_{\bar{\mathbf{x}};\theta}(\bar{\mathbf{x}};\theta)$ is the joint density of $\bar{\mathbf{x}}(\xi)$, which depends on a fixed but unknown parameter. Following [1 and 15-17] we have

$$\text{var}(\hat{\theta}) \geq -\frac{1}{E\{\partial^2 \ln f_{\mathbf{x};\theta}(\mathbf{x};\theta) / \partial \theta^2\}} \quad (4)$$

The function $\ln f_{\bar{\mathbf{x}};\theta}(\bar{\mathbf{x}};\theta)$ is well known as the “log likelihood” function of θ (LLF). Its maximum likelihood estimate can be obtained from the equation:

$$\frac{\partial \ln f_{\bar{\mathbf{x}};\theta}(\bar{\mathbf{x}};\theta)}{\partial \theta} = 0 \quad (5)$$

The first order of differential log likelihood function with respect to θ is called the maximum likelihood (ML) estimate.

It is noted that the value of about 1.5 is strongly recommended if there is no information about α due to the 2nd order simulations of the LLF for an alpha-stable [17].

3. WAVE-BASED BAYESIAN ESTIMATOR

If we take the probability density of θ as $p(\theta)$; and the posterior density function as $f(\theta | x_1, \dots, x_n)$, then the updated probability density function of θ is as follows:

$$\begin{aligned} f(\theta | x_1, \dots, x_n) &= \frac{f(\theta, x_1, \dots, x_n)}{f(x_1, \dots, x_n)} \\ &= \frac{p(\theta) f(x_1, \dots, x_n | \theta)}{\int f(x_1, \dots, x_n | \theta) p(\theta) d\theta} \end{aligned} \quad (6)$$

If we estimate the parameters of the prior distributions of the signal s and noise q components of the wavelet coefficients c , we may use the parameters to form the prior PDFs of $P_s(s)$ and $P_q(q)$, hence the input/output relationship can be established by the Bayesian estimator, namely, let input/output of the Bayesian estimator = BE , we have:

$$BE = \frac{\int P_q(q) P_s(s) ds}{\int P_q(q) P_s(s) ds} \quad (7)$$

$P_s(s)$ is the prior PDF of the signal component of the wavelet coefficients of the image and $P_q(q)$ is the PDF of the wavelet coefficients corresponding to the noise.

In order to be able to construct the Bayesian processor in (7), we must estimate the parameters of the prior distributions of the signal (s) and noise (q) components of the wavelet coefficients.

Then, we use the parameters to obtain the two prior PDFs $P_q(q)$ and $P_s(s)$ and the nonlinear input-output relationship BE .

Figure 1 shows the simulation results of input/output of BE with different α values for given $\gamma=25$ and noise of Poisson- ($\lambda=10$) and Gaussian- distributions ($\sigma=3.92, \mu=15$). It clearly shows that, for the given case, the curves with $\alpha=0.01, 0.1, 0.8, 1.5, 1.9,$ and 2.0 approximately correspond to the “hard”, “soft”, and “semisoft” functions respectively when compared with results in [8,18].

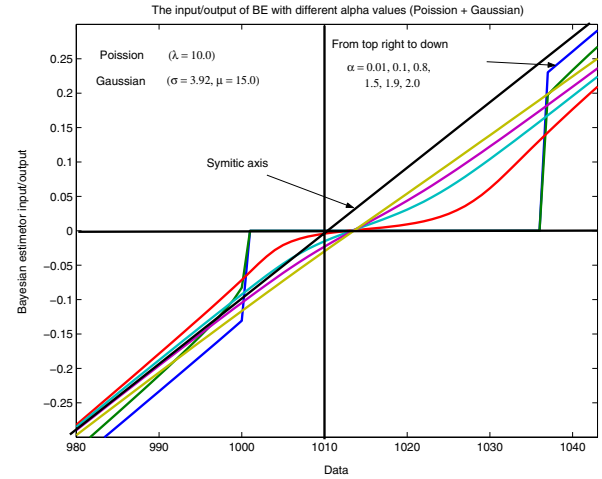


Figure 1: The input/output of BE with different α values for given $\gamma=25$, with noises of Poisson- ($\lambda=10$) and Gaussian- ($\sigma=3.92, \mu=15$) distributions.

It clearly shows that this input/output of BE is different from the case where the noise is purely Poisson [1], in particular for $\alpha=1.5, 1.9$ and 2.0 . Unlike the case contaminated by pure Gaussian noise [15-18] and Poisson noise [1], both parameters for Poisson and Gaussian distributions will affect the input/output of BE as shown in Figure 2, where the parameters are the same as that in Figure 1 except for the mean of Poisson distribution is equal to 20 rather than 10.

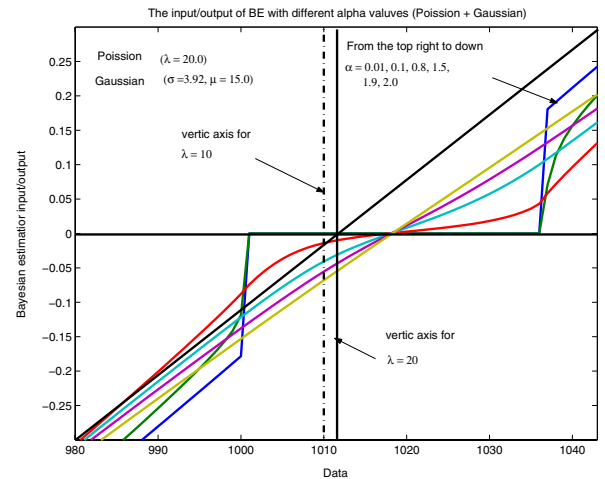


Figure 2: The mean of Poisson distribution affects BE . Here all parameters are the same as that in Figure 1 but the mean of Poisson distribution is 20.

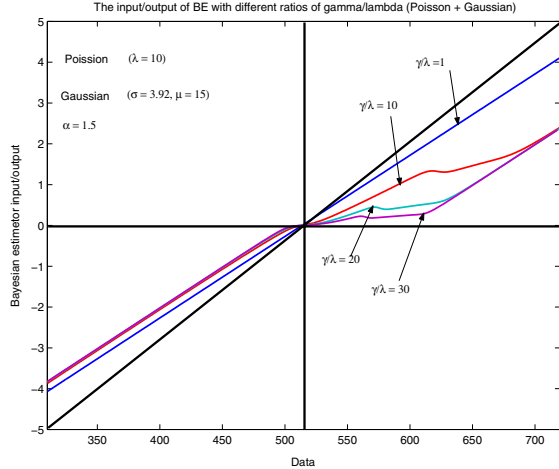


Figure 3: The input/output of BE with different ratios of γ/λ (with $\alpha = 1.5$, Poisson: $\lambda = 10$ and Gaussian: $\sigma = 3.92$, $\mu = 15$).

Figure 3 shows results if the parameters of Figure 1 are used varying the γ/λ ratio. In Figure 4 the parameter of Poisson and Gaussian as in Figure 3 are kept, but the mean of the Gaussian is changed from 15 to 25. These results are as expected since the noise sources are mixed together and both the parameters of the Poisson distribution (which is λ) and the parameters of the Gaussian distribution (which are σ and μ) will affect the input/output of BE. The two figures also show that γ will affect the output of BE, because $\gamma \in \mathbb{R}$ is the dispersion of the distribution. The ratios of 30 and 20 correspond to “semi-soft” and ratio of 10 corresponds to “soft” functions respectively.

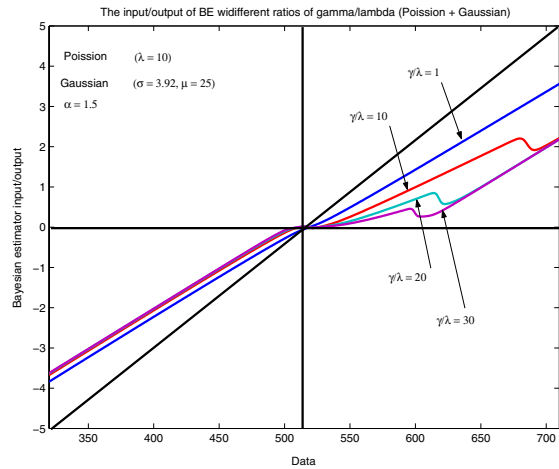


Figure 4: The input/output of BE with different ratios of γ/λ (with the same parameters as in Figure 3 except for $\mu = 25$).

4. SOME EXAMPLES

When noise is to be removed from an image using the wavelet-based Bayesian estimator, we normally have no information about the noise parameters of the image. The only information available is from experience in judging noise levels (if it is possible), which may become the basis of the denoising strategy. We take the parameters $\alpha = 1.5$, $\gamma/\lambda = 30$, in the two cases of BE with the Poisson ($\lambda = 10$) and Gaussian ($\sigma = 3.92$, $\mu = 15$) (Figure 8) and with Poisson ($\lambda = 10$) and Gaussian ($\sigma = 3.92$, $\mu = 25$) (Figure 9). It is important to note that the purpose of showing the

noise parameters here is only for comparison; knowing the parameters is not required for using this wavelet-based Bayesian estimator. In other words, the estimator did not know the noise parameters even we know them.



Figure 5: The Original Image “mountain”.

In order to compare the denoising results using this wavelet-based Bayesian estimator, we show the original colour image called “Mountain” in Figure 5, together with a copy contaminated by both Poisson and Gaussian noise in Figure 6. Figure 7 shows the image contaminated with three types of noise: Poisson, Gaussian and impulse noise. Note the differences between the two noisy images. The Harr mother wavelet was used for the wavelet based Bayesian estimator in this example.

We have used the parameter the ratio of the “signal” to “mean square error” as a criterion for a denoising method. Comparisons of other denoising results are in Table 1.



Figure 6: The Image contaminated with both Poisson and Gaussian noise.

Method	1	2	3	4	5
S/MSE	13.25	13.41	13.38	13.90	14.03

Table 1: Comparison of denoising results with BE in signal to mean square error (S/MSE) in dB. Here 1 = soft thresholding; 2 = Hard thresholding; 3 = Homomorphic Wiener; 4 = BE (Fig.3 with $\gamma/\lambda = 30$), 5 = BE (Fig.4 with $\gamma/\lambda = 30$).

5. CONCLUSION

The technique described using wavelet-based Bayesian estimators has been extended to treat signal-dependent noise obeying Poisson together with Gaussian distributions. This denoising technique does not require prior knowledge of the noise parameters or distribution. The statistician's Bayesian estimator theory has been extended to multi-noise removal for images. This

technique not only simplifies the selection of parameters but also, in some situations, provides more precise images than other methods.



Figure 7: The Image contaminated with Poisson, Gaussian and impulse noise.



Figure 8: The denoised image from the designed *BE* (with the parameters shown in Figure 3 and $\gamma/\lambda=30$).

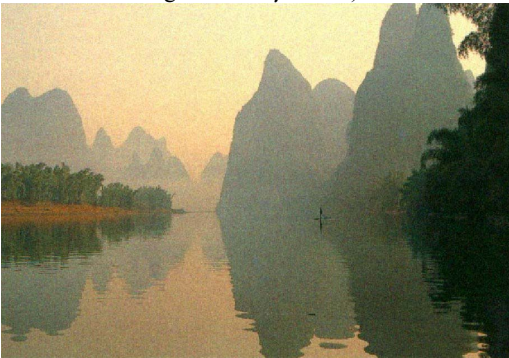


Figure 9: The denoised image from the designed *BE* (with the parameters shown in Figure 4 and $\gamma/\lambda=30$).

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