NOISES REMOVAL FOR IMAGES BY WAVELET-BASED BAYESIAN ESTIMATOR VIA LÉVY PROCESS ANALYSIS

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ABSTRACT

There are many noise sources for images. Images are, in many cases, degraded even before they are encoded. In our previous paper [1], we focused on Poisson noise. Unlike additive Gaussian noise, Poisson noise is signal-dependent and separating signal from noise is a difficult task. A wavelet-based maximum likelihood method for Bayesian estimator that recovers the signal component of the wavelet coefficients in original images by using an alpha-stable signal prior distribution is demonstrated to the Poisson noise removal. Current paper is to extend out previous results to more complex cases that noises comprised of compound Poisson and Gaussian via Lévy process analysis. As an example, an improved Bayesian estimator that is a natural extension of other wavelet denoising (soft and hard threshold methods) via a colour image is presented to illustrate our discussion, even though computers did not know the noise, this method works well.

1. INTRODUCTION

It is well known that noise degrades the performance of any image compression algorithm. In many cases it is, degraded even before they are encoded. There are many noise sources such as Poisson noise, Gaussian noise, impulse noise, salt and pepper noise, speckle noise, and etc. If we could have information about the noises, it would be helpful for us to design a good filter. We are going to extend our pervious research results [1, 14-16] to the noise sources consist of various major noise sources obeyed compound Poisson and Gaussian distributions via Lévy process analysis.

In 1963, Mandelbort [2] firstly pointed out the drawback of Brownian motion as a stock price model and proposed an exponential non-Gaussian Lévy process. After that Madan and Senata [3] have proposed a Lévy process with the variance of the normal distribution is gamma distributed as a model for the logarithm of stock prices. Barndor-Nielsen [4] proposed an exponential normal inverse Gaussian Lévy process. Bertoin [5] discussed Lévy processes in his book. The fundamental properties of Lévy process are established by the basic conceptual building blocks such as its characteristic function, infinitely divisible distribution, and the Lévy measure. In our current paper, the definitions and conditions of our investigations and discussions will follow the statements by Bertoin [5] unless indicated otherwise.

If Y'_is are independently identically distributed then Y would have a compound Poisson distribution with parameters $\Lambda = n\lambda_1$. As a consequence of the result the Lévy-Khintchine representation for the compound Poisson distribution is:

$$E[\exp\{i<\lambda,Y_1>\}] = E[\exp\{i\sum <\lambda,\Delta s>\}_{0\le s\le t}$$
$$= \exp\{-\int_{\Re^d}(1-\exp\{i<\lambda,x>\})\Lambda(dx)\} \quad (1)$$

Here, a finite measure Λ on $\mathscr{R}^d$ that gives no mass to the origin, let $\Delta = (\Delta t,\ t\ge 0)$ be a Poisson point process with characteristic measure Λ. The Lévy process $Y_t = \sum\Delta s$ with $(0\le s\le t)$ is well-defined. It was proved that the sum of independent Lévy processes is a Lévy process, thus, we can combine different "building blocks" to obtain a general characteristic function of the form same in the Lévy-Khintchine representation in the one-dimension case. For example, we can have the characteristic function of the stable (α,β,γ) distribution given by

$$\exp\{\gamma\,|\,\lambda\,|^{\alpha}\ [1-i\beta\mathrm{sgn}(\lambda)\tan(\pi\alpha/2)]\} \quad (2)$$

We are going to extend our method [1] for a stable (α,β,γ) distribution to the case contaminated by noises consisted of compound different Poisson distributions and together with a Gaussian distribution.

Several groups have discussed that wavelet subband coefficients have highly non-Gaussian statistics [7-16] and the general class of α-stable distributions has also been shown to accurately model heavy-tailed noise [10-11].

Wavelet transform as a powerful tool for recovering signals from noise has been of considerably interest [16-19]. In fact, wavelet theory combines many existing concepts into a global framework and hence becomes a powerful tool for several domains of application.

As mentioned by Achim *et al.* [21], there are two major drawbacks for thresholding. One is that choice of the threshold is always done in an ad hoc manner; another is that the specific distributions of the signal and noise may not be well matched at different scales.

In practice, the standard deviation can be readily estimated using the methods discussed in [11], [17]. Modelling the statistics of natural images is a challenging task because of the high dimensionality of the signal and the complexity of statistical structures that are prevalent.

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In this paper it is carefully discussed that a wavelet-based maximum likelihood for Bayesian estimator that recovers the signal component of the wavelet coefficients in original images from contaminated images by compound Poisson noises and Gaussian noise via using an alpha-stable signal prior distribution.

Without loss of generality, in order to focus on major parameters we take the parameter, called skewness, $\beta \in [-1,1]$ in above equation (2) to be unity. This is well known that the symmetric alpha-stable distribution.

2. WAVE-BASED BAYESIAN ESTIMATOR

Following our previous papers [1, 12-14], if we take the probability density of θ as $p(\theta)$; and the posterior density function as $f(\theta \mid x_1,...,x_n)$, then the updated probability density function of θ is as follows:

$$f(\theta \mid x_1,...,x_n) = \frac{f(\theta, x_1,...,x_n)}{f(x_1...,x_n)} = \frac{p(\theta) f(x_1,...,x_n \mid \theta)}{\int f(x_1,...,x_n \mid \theta) p(\theta) d\theta} \quad (3)$$

If we estimate the parameters of the prior distributions of the signal s and noise q components of the wavelet coefficients c, we may use the parameters to form the prior PDFs of $P_s(s)$ and $P_q(q)$, hence the input/output relationship can be established by the Bayesian estimator, namely, let input/output of the Bayesian estimator = BE, we have:

$$BE = \frac{\int P_q(q) P_s(s) s ds}{\int P_q(q) P_s(s) ds} \quad (4)$$

$P_s(s)$ is the prior PDF of the signal component of the wavelet coefficients of the ultrasound image and $P_q(q)$ is the PDF of the wavelet coefficients corresponding to the noise.

In order to be able to construct the Bayesian processor in (4), we can estimate the parameters of the prior distributions of the signal (s) and noise (q) components of the wavelet coefficients. Then, we use the parameters to obtain the two prior PDFs $P_q(q)$ and $P_s(s)$ and the nonlinear input-output relationship BE.

Figure 1 shows the simulation results of input/output of BE with different α values for given γ =25 and noises of Poisson- (λ=10) and Gaussian- distributions (σ = 3.92, μ =15). It clearly presented the fact that, for the given case, the curves with α = 0.01, 0.1, 0.8, 1.5, 1.9,and 2.0 are approximately corresponding to the "hard", "soft", and "semisoft" functions respectively in comparison with results in [9,16]. Unlike the case contaminated by pure Gaussian noise [12-14], the mean of Poisson noise plays a role in a BE as shown in Figure 2, where the parameters are the same as that in Figure 1 except for the mean of Poisson distribution is equal to 20 rather than 10. It is notice that not just axis shifting but also input/output converting curves are different in those two figures.

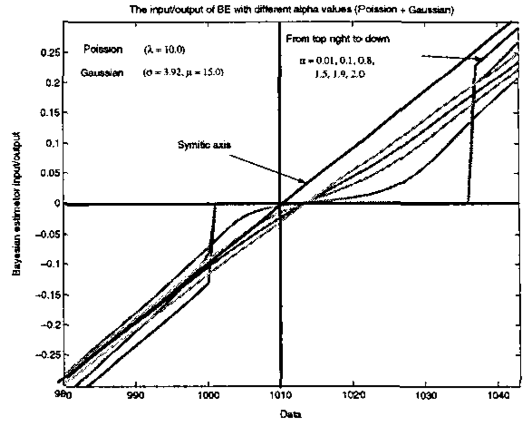

Figure 1: The input/output of BE with the contained noises of Poisson- (λ =10) and Gaussian- (σ =3.92, μ = 15) distributions.

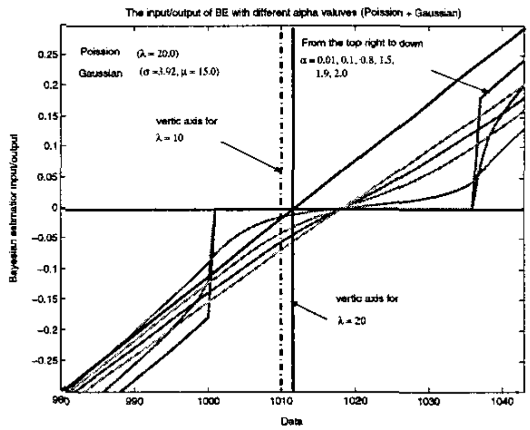

Figure 2: The input/output of BE with the same parameters in Figure 1 except for λ = 20 rather than λ =10.

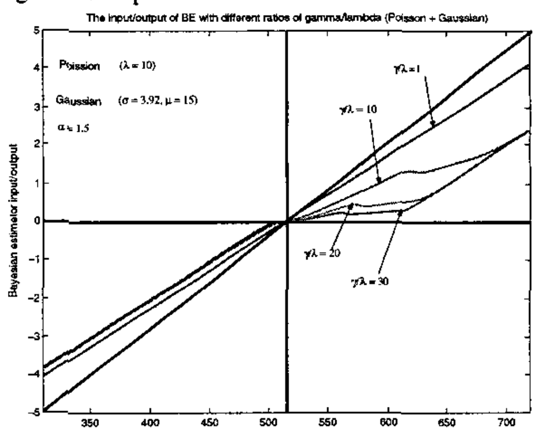

Figure 3: The input/output of BE with α = 1.5 and Poisson (λ =10) and Gaussian (σ = 3.92, μ = 15).

It is obviously that for the curves with fixed α value (=1.5) are significantly different with different γ values as shown in Figure 3 due to $\gamma \in R$ is the dispersion of the distribution. If we put two different Poisson noises together with a Gaussian noise, with $\alpha = 1.5$ as shown in Figure 4, with $\eta = (\lambda_1 + \lambda_2)/2$.

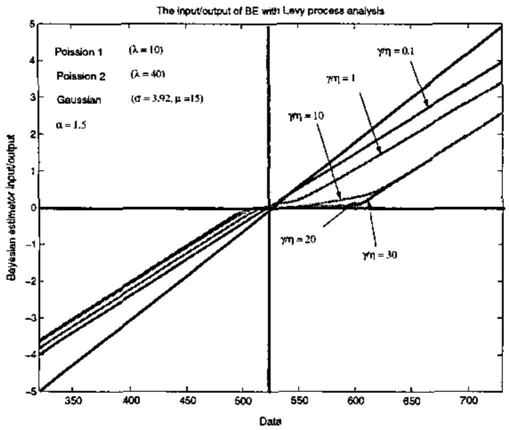

Figure 4: The input/output of *BE* with $\alpha = 1.5$ and Poisson 1 ($\lambda = 10$), Poisson 2 ($\lambda = 40$) and Gaussian ($\sigma = 3.92$, $\mu = 15$).

In Figure 4, it is clearly to show that the ratio of 20 and 30 are almost the same converting curve, a "hard"-like function. Figure 5 shows that the parameters of Gaussian will affect the input/out of *BE*, where keep the parameters the same as that in Figure 4 except for $\mu = 35$ rather than $\mu = 15$ for Gaussian distributions.

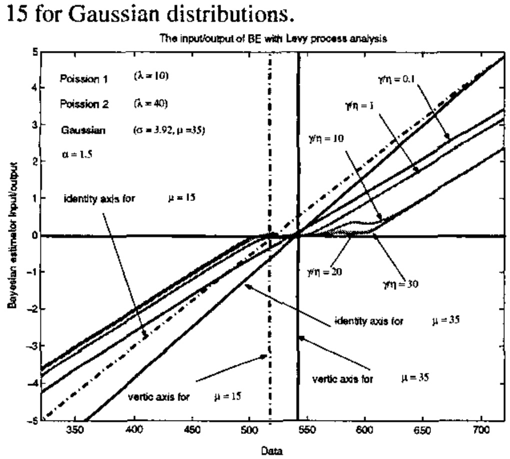

Figure 5: The input/output of BE with different ratios of γ/η with $\alpha = 1.5$, Poisson 1 ($\lambda = 10$), Poisson 2 ($\lambda = 40$) and Gaussian ($\sigma = 3.92$, $\mu = 35$).

3. SOME EXAMPLES

When we make measurements, we have no information about the noise values of the image we obtained. We take the parameters $\alpha = 1.5$, γ/mean = 20, in the case of *BE* with the Poisson 1 ($\lambda = 10$), Poisson ($\lambda = 40$), Gaussian ($\sigma = 3.92$, $\mu = 15$) distributions (refer to Figure 4). In order to compare, we show the original image called "mountain" in Figure 6, together with its contaminated one by Poisson noise in Figure 7 and the one with its noised compound Poisson distributions in Figure 8. In Figure 9 the contaminated by compound Poisson and Gaussian noises is shown. The Harr mother wavelet was used for this example. The output of denoised image from *BE* is shown in Figure 10.

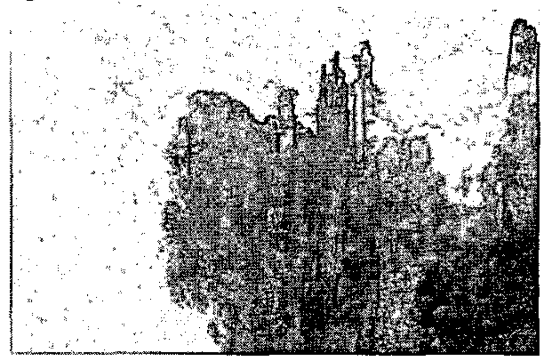

Figure 6: An Image of the "mountain".

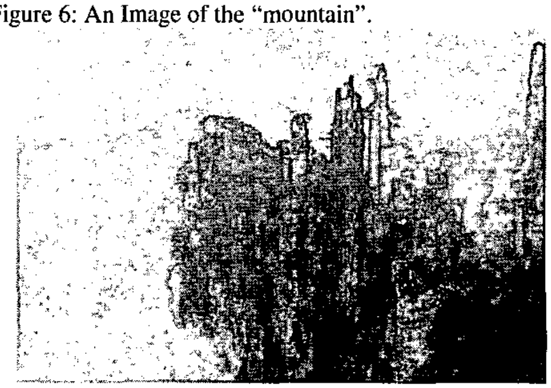

Figure 7: The contaminated Figure 6 by Poisson noise

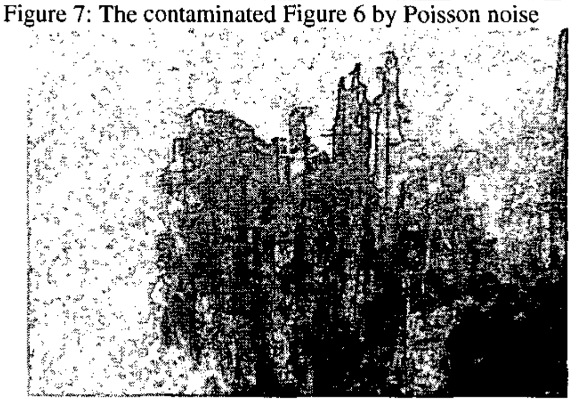

Figure 8: The contaminated Figure 6 by compound Poisson noises.

Comparisons of other denoising results are in Table 1.

Method	1	2	3	4
S/MSE	13.85	13.92	13.71	14.17

Table 1: Comparison of denoising results with *BE* in signal to mean square error (*S/MSE*) in dB. Here 1 = soft

thresholding; 2 = Hard thresholding; 3 = Homomorphic Wiener; 4 = BE ($\gamma/\eta = 20$ in Fig.4).

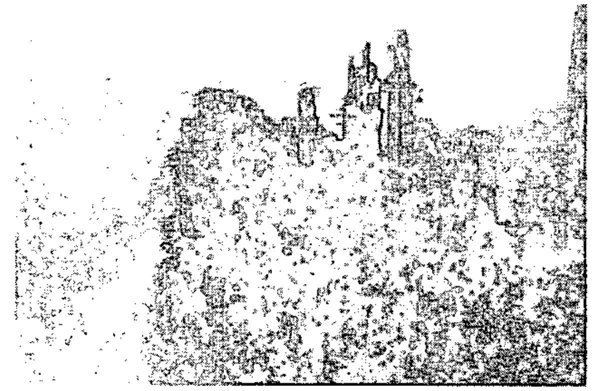

Figure 9: The contaminated Figure 6 by compound Poisson noises and Gaussian noise.

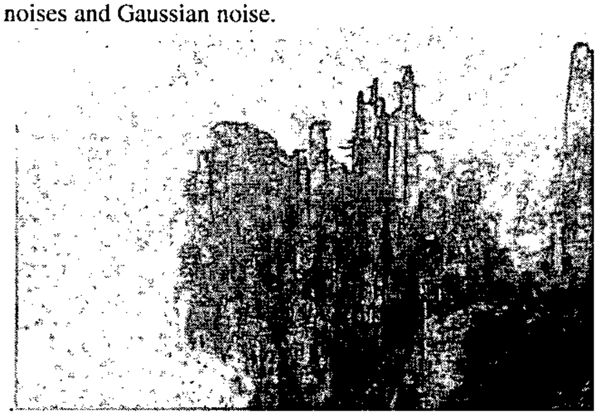

Figure 10: The denoised image from the designed BE with the parameters shown in Figure 4 with $\gamma/\eta = 20$.

4. CONCLUSION

The technique uses the wavelet-based Bayesian estimator has been extended to the signal-dependent noise obeying Poisson distribution and Gaussian noise via Lévy Process analysis. The statistician's Bayesian estimator theory is not only to simplify the selection of parameters and also in some situations to provide more precise images than other methods.

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6. REFERENCES

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