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The Influence of Spatial Visualization Training on Students' Spatial Reasoning and Mathematics Performance

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ABSTRACT
Over three decades of research has shown that spatial reasoning and mathematics performance are highly correlated. Spatial visualization, in particular, has been found to predict mathematics performance in primary and middle school children. This research sought to determine the effectiveness of a spatial visualization intervention program on increasing student spatial reasoning and mathematics performance. Participants were 327 students from 17 classrooms across ten schools with nine experimental and eight control classes. The intervention program was delivered over a three-week period by classroom teachers, while the control classes received standard mathematics instruction. When compared to the control group, participants in the intervention group improved significantly on their spatial reasoning performance, and specifically on spatial visualization and spatial orientation. The intervention group also significantly improved on their mathematics test performance, with those in the intervention group outperforming their control group peers on geometry and word problems but not on mathematics questions requiring the decoding of graphics (non-geometry graphics tasks). These results add to evidence that a spatial reasoning enrichment program implemented by teachers in their own classrooms can enhance both spatial reasoning and mathematics performance. Moreover, the study provides new insights about the aspects of mathematics performance that are most affected by spatial visualization training.

Spatial reasoning has been consistently linked to student performance in Science, Technology, Engineering and Mathematics (STEM) subjects in school (Lean & Clements, 1981; Lowrie & Diezmann, 2007) and the likelihood of engagement with a STEM career (Kell, Lubinski, Benbow, & Steiger, 2013; Wai, Lubinski, & Benbow, 2009). Although school systems across the world have a heightened awareness of promoting success in STEM, the development of spatial reasoning is not typically included as an explicit goal in school curricula (National Research Council, 2006). Given mounting evidence for a strong association between spatial reasoning and STEM success (e.g. Wai et al., 2009) researchers have suggested that early spatial intervention may increase students' spatial competencies, so they are not overwhelmed by spatial demands of STEM learning in the later years of education (Newcombe, 2016; Uttal et al., 2013). Spatial reasoning involves the understanding

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of three related properties: (1) an awareness of *space* itself, such as distance and dimensions; (2) the *representation* of spatial information (internally in the mind and externally in graphics such as diagrams and maps); and (3) the *reasoning* involved in interpreting and manipulating spatial information for problem solving and decision making (Carroll, 1993; National Research Council, 2006). To this point, spatial reasoning can be regarded as a general meta term for a range of spatial skills, including those measured by psychometric tests of spatial abilities or skills. Such spatial skills include spatial visualization, mental rotation and spatial orientation (Ramful, Lowrie, & Logan, 2017).

Spatial visualization is defined as the ability or skill drawn upon to mentally transform or manipulate spatial properties of an object. Spatial visualization is often measured with instruments such as the Paper Folding Test and the Form Board Test, (Ekstrom, French, Harman, & Dermen, 1976) which require participants to imagine a series of spatial transformations. It has been found to predict mathematics performance (Shea, Lubinski, & Benbow, 2001; Tversky, 2011; Uttal et al., 2013; Wei, Yuan, Chen, & Zhou, 2012), at least in part because higher level mathematics performance involves the ability to imagine and visualize (Battista, 1990; Battista, Wheatley, & Talsma, 1982).

Mental rotation is a specific type of object-based transformation, often separated in factor analytic studies from measures of spatial visualization (Hegarty & Waller, 2005; Mix & Cheng, 2012). In mental rotation tasks the 2D shape or 3D object remains intact during its movement through imagined space (Sorby, 1999). Within mathematics, mental rotation has been linked with early mathematical development due to shared processes with constructing and manipulating mental models (Mix et al., 2016). Spatial orientation, refers to the ability to reorient oneself in space and involves the process of mapping spatial relations at different scales, and from different perspectives and locations within the environment. It is believed to be distinct from object-based transformations (Mix & Cheng, 2012; Uttal et al., 2013).

Malleability of spatial reasoning

There is increasing evidence that spatial training generally improves performance on spatial measures, indicating that spatial skills are malleable (Uttal et al., 2013). In an extensive meta-analysis of 217 studies examining training of spatial skills, Uttal et al. (2013) found substantial evidence that spatial skills can be improved with training, with an average effect size of .47. Some of these studies indicated that effects of training transferred to untrained spatial skills. Studies have also found greater improvements in performance of students with lower initial levels of spatial reasoning (Cheng & Mix, 2014; David, 2012; Lowrie, Logan, & Ramful, 2017; Mix et al., 2016; Taylor & Hutton, 2013; Wei et al., 2012).

Existing studies of the effects of spatial training programs vary on a number of attributes, including type of training, the specific spatial skills targeted, the duration of the training program, the age of participants, and who delivers the instruction. Types of training include instructional courses targeting a range of spatial skills (e.g., Sorby, 1999), video-game playing (Feng, Spence, & Pratt, 2007; Sanchez, 2012), and practicing of specific spatial skills, often in the context of a psychology laboratory (e.g., Wright, Thompson, Ganis, Newcombe, & Kosslyn, 2008). Most training programs have targeted particular spatial skills, as identified in the spatial reasoning literature, including mental rotation (e.g., Blüchel, Lehmann, Kellner, & Jansen, 2013) and spatial visualization (Ben-Chaim, Lappan, & Houang, 1988;

Duesbury & O'Neil, 1996). A majority of training studies have been laboratory-based studies with short interventions of an hour or less, while some training durations have been as long as a full semester-length course (Sorby, 1999). Although training programs have been implemented with children from the early years of school through to adults, most programs have concentrated on undergraduate students (Uttal et al., 2013). Finally, most training programs have been implemented by members of a research team, with the students' own classroom instructor rarely involved. Studies that examine the effectiveness of intervention programs to improve spatial reasoning within classroom contexts have been rare.

Although there is now good evidence that spatial skills can be trained, spatial training has become a subject of some debate in the literature on improving STEM performance, because there is little evidence to date that this training transfers to performance in mathematics and other STEM domains. Stieff and Uttal (2015) argued that to provide evidence for such transfer, a study must demonstrate that spatial training improves spatial skills and that the resulting improvement in spatial reasoning leads to improvements in STEM outcomes. Studies that trained specific skills, such as Hawes, Moss, Caswell, and Poliszczuk (2015) and Xu and LeFevre (2016), demonstrated improvements in specific spatial reasoning skills which generally did not transfer to mathematics. By way of example, Hawes et al. (2015) compared 6 weeks of training in mental rotation to literacy training and found that the training improved mental rotation performance but did not transfer to mathematics. Similarly, Xu and LeFevre (2016) trained students on decomposition of shapes and found transfer to tasks that involved mental spatial transformations but not to mathematics.

Spatial reasoning constructs and mathematics performance

This section examines the effects of spatial reasoning training on both spatial skills and mathematics performance. Mix et al. (2016) assessed relations between a range of spatial and mathematics skills in 854 children ranging from five to thirteen years of age. They found that mental rotation was the best predictor of mathematics performance at kindergarten, while visual-spatial working memory was more important by Grade 6. The link between spatial reasoning and mathematics is evident, and this appears particularly true for spatial visualization (Mix & Cheng, 2012; Mix et al., 2016). Given this evidence, there is cause to consider the viability of spatial visualization training for improving mathematics performance.

In relation to effects of spatial training on mathematics performance, three studies have demonstrated potential (Cheng & Mix, 2014; Hawes, Moss, Caswell, Naqvi, & MacKinnon, 2017; Lowrie et al., 2017). Cheng and Mix (2014) implemented an intervention program to improve mental rotation and mathematics performance in fifty-eight 6- to 8-year olds. Their results suggested a causal link between spatial and mathematical reasoning, in that children who participated in the intervention improved not just on mental rotation tasks, but also outperformed their peers in one post-intervention assessment of mathematics – specifically missing-term problems. The intervention in this study was a single session, implemented by a member of the research team, however, Cai et al. (2017) argued that to truly impact classroom practice it is vital for research to focus on classroom implementation.

Hawes et al. (2017) developed a spatial intervention program (for 4–7 year olds) for the first three years of formal schooling. Teachers implementing the intervention undertook in-person professional development (PD) before implementation and throughout the intervention, which comprised five hour-long geometry lessons and a series of spatial challenges that engaged participants in spatial visualization activities. Students in the experimental group spent approximately 47 hours on these activities across the 32-week intervention. Compared to an active control group, the intervention group had statistically significant gains on all three measures of spatial reasoning including spatial language, visual-spatial geometry and mental rotation. Despite the addition of the five focused geometry lessons during the intervention, improvement in mathematics was limited to one of three measures involving symbolic number comparison. There were no gains in a nonsymbolic comparison test or a measure of number knowledge.

Lowrie et al. (2017) conducted a 10-week intervention program to enhance students' spatial reasoning in their own classroom with instruction by their classroom teacher. Teachers participated in a week-long professional development program that introduced them to spatial reasoning and then teachers collaborated on the curriculum development with researchers. The intervention included three weeks of instruction each on mental rotation, spatial orientation and spatial visualization, followed by a week of integration of these spatial skills. Control groups received mathematics instruction as usual. Participants in the intervention program significantly improved and outperformed those in the control group on spatial reasoning with a medium effect size (Cohen's d = .54) and this improvement generalized to performance on a mathematics test (d = .40) post intervention (Lowrie et al., 2017). The test comprised items involving number, geometry and measurement content.

Effects of intervention length

Given that the duration of spatial training has varied considerably across studies, a question arises as to how long a spatial intervention needs to be to lead to meaningful learning. In contrast to the single intervention session implemented by Cheng and Mix (2014), the sustained ten-week intervention program (Lowrie et al., 2017) led to broader and more comprehensive improvement in mathematics concepts (including concepts associated with measurement and geometry). This raises the question of whether a shorter intervention program, focusing on some but not all spatial components, could have significant effects on spatial and mathematics performance. Here, we study the effects of a 3-week spatial intervention on spatial and mathematics performance. The intervention was based on that of Lowrie et al. (2017) but focused on spatial visualization only. It also differed from the original program in that computer-based resources were used in addition to hands-on resources. Finally, the intervention was conducted in more classrooms and with larger samples of students, allowing for the data analysis using multi-level (hierarchical) modeling.

Effects of spatial training on different aspects of mathematics

A final question raised by previous studies is which aspects of mathematics training are affected by spatial training. From a theoretical perspective, there has been a long-held view that students develop spatial representations both in the mind (visual-spatial images) and on paper (e.g., diagrams) to process mathematics information (Krutetskii, 1976; Presmeg, 1986; Zhang & Lin, 2017). For example, students report "seeing" numbers along a mental number line and enacting these representations by identifying smaller numbers faster with their left hand and larger ones with their right hand (Dehaene, Bossini, & Giraux, 1993; Hoffmann, Hornung, Martin, & Schiltz, 2013). Resnick (1992) proposed that use of spatial reasoning to identify, distinguish between, and collect groups of objects and concepts notably contributes to mathematical ability in early childhood and beyond (see also, Kahneman, Treisman, & Gibbs, 1992; Mix & Cheng, 2012; Zhang & Lin, 2017). Similarly, proponents of embodiment theory argue that abstract mental representations of mathematical concepts are formed on the basis of concrete physical actions, interactions, and experiences (Barsalou, 2008; Clark, 1997; Mix & Cheng, 2012). These theories suggest that the effects of spatial training on mathematics performance might be general. But in fact, Cheng and Mix (2014) found these effects to be quite specific. In their study, mental rotation training improved solution of missing term problems relative to a control group, but the groups did not differ in improvement on multidigit calculation or number fact questions.

Here we examine the effects of spatial visualization on three sub-categories of mathematics problems; (1) geometry problems, (2) word problems and (3) problems that involve decoding graphics (non-geometry). We hypothesized that spatial visualization would improve geometry performance since geometry is a branch of mathematics that requires the ability to transform and manipulate spatial properties (Clements & Battista, 1992). There is also evidence that spatial and visual representations of geometric understandings provide students with supportive scaffolds to access more advanced geometric concepts (Mamolo, Ruttenberg-Rozen, & Whiteley, 2015).

We might expect spatial training to improve performance on mathematics word problems because these problems are often solved by creating an internal spatial representation of the information in the problem (Lowrie & Clements, 2001). These internal representations might then be externalized as diagrams or other spatial representations. Hegarty and Kozhevnikov (1999) classified two types of visuo-spatial representations created by children in mathematics problem solving: (1) schematic representations encoding spatial relations described in a problem, and (2) pictorial representations of the visual appearance of items described. There was a strong association between use of schematic representations and spatial visualization, suggesting that spatial reasoning is related to the ability to construct effective visual-spatial representations that encode the essential information necessary for solving a problem. While their study was not a training study, it raises questions about whether improvements in spatial visualization may prompt improvements in the use of spatial representations in mathematics problem solving.

Finally, we examined whether spatial visualization training affected performance in mathematics problems that involve interpreting graphics. Tasks that have embedded graphics, such as diagrams and graphs require students to decode these graphics, which are often highly conventional. In contrast to solving word problems, where a spatial representation might be created by the problem solver, decoding processes involved in tasks such as interpreting a bar graph or reading a map, involve knowing the conventions of a representation that one did not create oneself (Pape & Tchoshanov, 2001). To date, the extent to which improvements in spatial reasoning enhance students' capacity to successfully decode common graphical representations used in mathematics as opposed to their capacity to encode information spatially (into a new spatial representation) has not been tested.

Theoretical basis of the spatial visualization training

Spatial visualization activities in the intervention were framed around a pedagogical framework that encourages concept development through various embodied experiences. The lessons were embedded within the Experience-Language-Pictorial-Symbolic-Application (ELPSA) learning framework (Lowrie, Logan, Harris, & Hegarty, 2018; Lowrie & Patahuddin, 2015). The first component of the ELPSA learning framework (Experience) draws on the knowledge that students possess. Typically, students are asked to talk about the topic in relation to their own experiences, whether within the curriculum or out-of-school contexts. The second component (Language) outlines how terminology is used to promote understanding. In this stage, appropriate terminology is used to encourage students to describe their thinking in ways that reinforce their knowledge and promote discourse with others. The third component (Pictorial) positions learning around manipulatives and other spatially embodied information (e.g., mental visualizations) to connect representations to conceptual understandings. Such representations could be constructed by the teacher (e.g., shared resources and artifacts) or students (including drawing diagrams or visualizing). The fourth component (Symbolic) is aligned to the formalization of ideas or concepts. This stage draws on students' capacity to represent, construct, and manipulate analytic information with flexibility and a degree of fluency (Stieff, 2007). The final component of the learning framework (Application) highlights how symbolic understanding can be applied to new situations. This is evident in students' ability to transfer their knowledge to novel situations.

The implementation of the lessons within the ELPSA framework was accompanied by an emphasis on the Pictorial component. A student-focused approach to the Pictorial component consisted of tasking students to visualize a problem, attempt to predict outcomes through visual representation, experiment, physical manipulation of stimuli, and to finally check outcomes against the initial visualization and representation. Participants were provided with ample opportunities to attempt to visualize, represent their thinking and engage in lessons. This Pictorial component of the framework also provided opportunities for engagement with digital tools that encouraged students to practice mental manipulations of spatial relations. The digital representations complemented the hands-on approaches, with the advantage of allowing students to progress through activities at their own pace.

The present study

This investigation was concerned with the effectiveness of a three-week version of a digitally-enhanced spatial visualization intervention program for improving: (a) spatial reasoning as measured by spatial skills and (b) mathematics performance of students from ten elementary schools. Classrooms in these schools received either the spatial intervention or acted as business-as-usual controls[1]. Based on previous research (summarized by

[1]Due to ethical restrictions set by governing educational jurisdictions, random assignment was not possible. Pre-test comparisons were conducted to ensure group equivalence before the intervention.

Uttal et al., 2013) we hypothesized that the intervention program would result in improved spatial reasoning. Based on the known correlation between spatial and mathematics performance and previous research indicating that spatial training improves mathematics performance (Cheng & Mix, 2014; Lowrie et al., 2017), we also hypothesized that students in the intervention group would improve in mathematics performance more than the control group over the course of the intervention period. In addition, we examined the effects of spatial training on three aspects of mathematics, namely geometry, word, and graphics problems.

Method

Participants

A total of 327 grade 5 and 6 students (age range 10– 12) from ten primary schools in rural and regional NSW, Australia participated in the study. Students (93 males, 84 females; mean age = 10.59 years) from nine classes participated in the intervention condition and students from eight classes (79 males, 71 females; mean age = 10.84 years) participated in the control condition. More than 85% of the students had English as their first language. In the absence of random allocation, schools were matched on sociodemographic information.

All schools in the study were drawn from average sociodemographic areas. In Australia, the socioeconomic advantage of a school is measured by the Index of Community Socio-Educational Advantage (ICSEA) scale. A score (Mean = 1000, S.D = 100) is produced for each school, based on Australian Bureau of Statistics (ABS) data, school location, and the proportion of Indigenous students enrolled in the school as well as data on parent's self-reported income, qualifications and occupation. Thus, a value on the index corresponds to the average level of educational advantage of the school's student population relative to those of other schools. The ICSEA scores, ranging from 987 to 1066 for the intervention schools and 1007 to 1051 for the control schools, were not significantly different between intervention and control, $t(8) = .09$, $p = .77$

Spatial reasoning instrument

The spatial training program was assessed in the present study by an established measure of spatial reasoning (Spatial Reasoning Instrument; SRI) developed for this age group and population (Ramful et al., 2017), that is, elementary-school students aged between 9 and 12. The SRI provides broad coverage of three spatial skills (sub-constructs of spatial reasoning) that have been well established in the research literature (Linn & Petersen, 1985; Lohman, 1979; McGee, 1979), namely mental rotation, spatial orientation and spatial visualization. The three subscales have strong correlations with measures of these constructs in the cognitive psychology literature (Ramful et al., 2017). The paper-and-pencil SRI consisted of 30 multiple-choice items. Score on the SRI was the total number of correct items for each participant; unanswered items were assigned a score of 0. Examples of the items are presented in Figure 1.

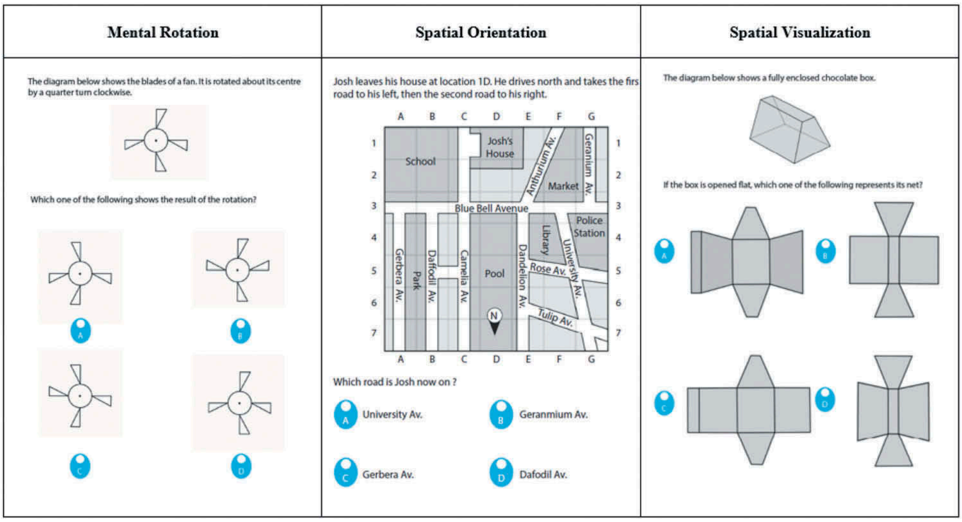


Figure 1. Sample items from the SRI (Ramful et al., 2017).

Mathematics test

Students' mathematics performance before and after the intervention program was assessed using a mathematics test developed from items released by Australia's National Assessment Program (NAPLAN) Numeracy test. The items were drawn from the Grade 5 and Grade 7 NAPLAN tests, consequently the items were age appropriate. All items selected for the test were analyzed by the Australian Curriculum, Assessment and Reporting Authority (ACARA)[2] for item reliability and content identification. The 20 multiple-choice and short-answer questions contained assessment items associated with geometry and measurement, number and algebra and probability. Items required application of mathematics knowledge instead of calculations or drill-and-practice procedures traditionally associated with questions on math assessments. The items were classified in terms of task representation, which included traditional word problems, geometry-based tasks and other graphics tasks that contained non-geometry mathematics. Three researchers independently categorized the assessment items based on the three types of representations, reporting a high Inter-Rater Reliability (Cronbach Alpha = .99). There was a relatively even distribution of geometry-based graphic problems (n = 7), word number problems (n = 7) and non-geometry graphics problems (n = 6). These graphics problems included content areas such as probability, measurement and algebra. See Figures 2–4 for examples of a geometry, word and non-geometry graphic problems, respectively.

Questions on the mathematics test were given a score of 1 for correct or 0 for incorrect, therefore the highest potential score was 20. Table 1 presents the Cronbach's alpha for reliability estimates, for pre- and post-tests, for both the spatial reasoning and mathematics instruments. Test-retest reliability measures for tests and sub-components are also displayed, with alpha levels of .87 and .91 for the spatial and math tests respectively.

[2]ACARA is the national testing service in Australia and has similar functions and roles to that of the Educational Testing Service (ETS) in the United States.

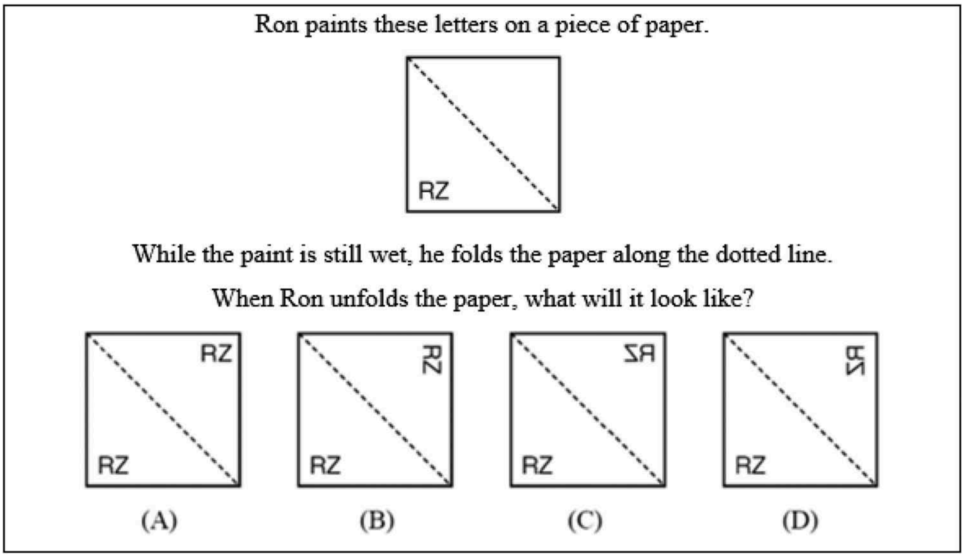


Figure 2. A geometry problem associated with diagonal symmetry.

Ben puts 7 flowers in each of 8 vases.

He has 3 flowers left over.

Ben wants to put 9 flowers in each vase.

How many **more** flowers does he need?

Figure 3. A mathematics word problem associated with number sense.

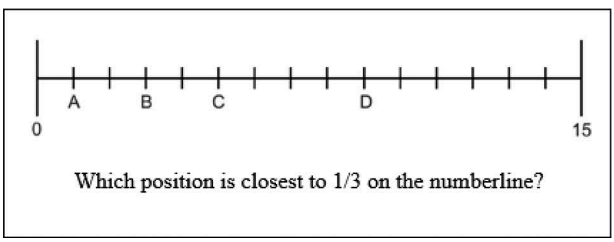


Figure 4. A non-geometry graphics-based problem (number line) focused on measurement sense.

Research design

An expression of interest was sent to schools from the educational jurisdiction to recruit teachers. Teachers from participating schools self-selected to the intervention or control groups (each school was represented only in one condition). The study ran in the fall term of the 2017 school year. The intervention group followed the prescribed program in lieu of geometry and measurement lessons while the control group continued with business as usual as outlined in the Australian Curriculum.

Table 1. Internal consistency (α) and test-retest reliability of test instrument pretests and posttests.

	Pretest		Posttest		Test-retest
Test Instruments	Items	α	Items	α	α
Math Test (n = 327)	20	0.73	20	0.71	0.91
Geometry	7	0.72	6	0.71	0.75
Word	7	0.72	7	0.71	0.78
Non-Geometry	6	0.74	7	0.73	0.78
SRI Test (n = 327)	30	0.77	30	0.81	0.87
Spatial Orientation	10	0.70	10	0.70	0.82
Mental Rotation	10	0.71	10	0.76	0.75
Spatial Visualization	10	0.65	10	0.71	0.75

In each class, testing took place in the classroom during regularly scheduled school hours. The Mathematics test and SRI were administered to whole classes by the classroom teacher. The tests were untimed but were completed by all students within 90 minutes. After a brief introduction each child worked on an individual test booklet. Testing was completed within the week prior to the commencement of the three-week intervention (pretest) and within a week of its completion (posttest) for both the control and intervention classes.

Professional development workshop

Teachers in the intervention condition participated in a university-led two-day Professional Development (PD) workshop (totaling 10 hours) to introduce them to the Spatial Reasoning program within the two weeks preceding commencement of the three-week intervention. During the workshop, classroom teachers were introduced to the spatial reasoning constructs and the lesson plans developed for the spatial visualization activities. At the completion of the workshop, the intervention program teachers were equipped with the detailed lesson plans and teaching materials (including concrete manipulatives). Digital resources were also provided, which included appropriate visualization games accessed from Google Play (e.g. Symmetria, Illuminations and Geogebra, see Table 3). All apps were free or were purchased at a nominal cost to exclude advertisements. The teachers were provided with class sets of Samsung tablets that were made available to each intervention school for the program. The teachers were encouraged to use these apps most commonly at the Pictorial or Application phases of instruction. Recall, these components of the framework provided opportunities for students to represent objects in multiple ways (Pictorial) or apply symbolic understandings to new situations (Application).

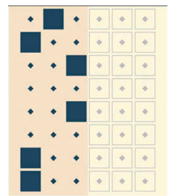
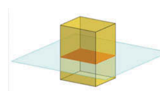
Content of the spatial reasoning program

The intervention was implemented in whole classes over three weeks in 2017 during twice weekly 60-minute class periods. The lessons were designed around the ELPSA pedagogical framework. A diverse range of spatial visualization activities and topics (such as reflection, symmetry and paper-folding, see Table 2) were chosen so that children could begin to develop more flexible approaches to encountering novel problem contexts (Reys, Lindquist, Lambdin, & Smith, 2009). To this point, the lessons sought to:

Table 2. Structure and nature of the intervention lessons.

Week	Specific topics	Spatial terminology	Learning intentions
1a	Reflections	2D shapes, translations, reflections, rotations	Complete reflections of 2D shape/3D objects with/without mirrors; 2D shapes over vertical/horizontal/inclined axes
1b	Symmetry	Line of symmetry, rotational symmetry	Create images/drawing show understanding of symmetry; symmetry in environment
2a	Paper folding and cutting	Repeated patterns;	Visualize folded and cut patterns; encode procedures for paper folding
2b	Nets of solids	Edge, faces, sides, shape, vertices, cubes.	Recognize nets that form a cube; 2D representations of 3D objects
3a	Hidden blocks	Layers, arrays	Count hidden blocks in built objects; recognize patterns.
3b	Cross-sections 3-D objects	Cross sections, cuts, slice.	Describe 2D cross sections of 3D objects, Create 3D objects and dissect

Table 3. Digital tools used in the lessons.

Week	Game Name	Game Intention	Example
1	Symmetria: Path to Perfection	Students were required to create symmetry in-app using patterns provided.	
2	Illuminations – Cube Nets	Students were presented with a variety of nets and had to predict whether or not they would form a cube. App features allowed demonstration of the folded net.	
3	Geogebra	Students used the app to explore the 2D shapes created with the cross sections of 3D objects.	

(a) find out what the students knew about the topic and how the topic related to personal experiences (experience);
(b) encouraged the teacher and the students to use explicit language in order to reinforce spatial content, including providing teachers with specific questions to pose (language);
(c) represent concepts in different ways, including the purposeful use of concrete materials, gesture, student encoding and the digital games (pictorial);
(d) encourage students to represent spatial understandings with more fluency, such as recognizing patterns that reduced the burden on visualization in the mind's eye (symbolic); and
(e) apply concepts to new situations or use the digital games to reinforce concepts (application).

Table 2 provides a summary of the learning activities presented in the intervention program. The table also includes the specific terminology addressed in the lessons, as well as the spatial intent of the learning experiences. In week one, the spatial visualization development focused on the interpretation and manipulation of 2D shapes. The visualizations included distinguishing between rotations and reflections, before progression to rotational symmetry. Week 2 focused on the relationship between 2D shapes and 3D objects, including visualization of the nets of cubes. Students were encouraged to mentally transform and manipulate from 2D to 3D and from 3D to 2D. Week 3 involved visualization of 3D objects, including cross sections of objects. The intervention replaced the measurement and geometry units that would usually be taught from the Australian Curriculum.

The intervention cohort was also asked to use digital tools as part of the intervention program; typically incorporated into the application (A) component of the ELPSA cycle. These games were downloaded onto tablets, with the intent of providing opportunities for students to practice skills acquired during the intervention lessons. The teachers were encouraged to limit individual's engagement on the devices to 30 minutes per week, given issues associated with screen time in schools. Moreover, the teachers were asked to limit other digital experiences during the three-week trial. Table 3 provides examples of the digital tools utilized during the intervention.

Control group

The control groups' learning activities were drawn from the respective teachers' curriculum program (ACARA, 2015). In Australia, the curriculum outlines the necessary content to be taught for each age group, but the school and classroom teacher determines the structure of the lessons. The prescribed content includes geometry and measurement, number and algebra, and statistics and probability. The business-as-usual practices in classrooms tend to involve four hours of mathematics instruction per week. Typically, teachers devote one third of their time to the geometry and measurement strand (80 mins per week). The development of students' spatial reasoning skills would be typically covered in the geometry strand of the mathematics curriculum, in particular, content associated with 2D shape and 3D objects, location and transformation, use of grid references on maps and the introduction of the Cartesian coordinate system. In the business-as-usual context, student's engagement with digital tools most likely includes drill-and-practice games aligned to the mathematics curriculum (e.g., Mathletics; 3P Learning, 2017).

Fidelity monitoring

In terms of lesson implementation, a fidelity protocol was developed to determine the adherence to lesson structure, evidence of the pedagogical approach (ELPSA) and completion of program content. Fidelity monitoring was carried out in three ways, namely: (1) a site visit by members of the research team to observe lesson implementation; (2) reflection activities by classroom teachers after completion of lessons; and (3) post-analysis of student workbooks to track the level of content completion.

For the site observations, two sets of questions were developed to measure the adherence to the program and evidence of the pedagogical approach through a Likert-type scale ranging from 0 (not observed) to 4 (high). The sets of questions were associated with lesson plan structure and the extent to which the ELPSA components were utilized. Only one observation per class was feasible given the vast distances between the schools and the university – the closest school site was 99 miles North-West of the university while the furthest site was 141 miles South-East of the university. Three experienced teacher-researchers from the team visited the sites during the three-week intervention, sharing the workload such that two raters only visited each site. Each rater coded the lesson independently with observations recorded for all but the first lesson. Internal consistency across the rater scores for the intervention classes was high (Cronbach's Alpha = .81).

All teachers completed a reflection activity after administering either the first or second intervention lesson. The activity required the classroom teachers to self-report on the extent to which they adhered to the ELSPA pedagogical approach, coverage of content and views on their student engagement. The format of the reflection activity comprised the 0–4 Likert-type scale statement as well as open-ended questions. The following statements from classroom teachers are drawn from the open-ended questions, highlighting program adherence.

> I followed lessons as prescribed. We use the framework in all Maths lessons. Talented student tends to go straight to symbolic but has had to slow down to express thinking with language and pictures. Most of the class enjoy experience and language and pictorial tasks. [Teacher A]
> Yes, I made a poster explaining ELPSA with images, so the students would understand the process. During and after lessons I referred to our poster and highlighted which parts of the framework we worked in during the lesson. Some of the students are now telling me what parts they are working in, so I hope by the end of the 3 weeks I am hoping that level of recognition increases. [Teacher C]
> Yes, I keep coming back to the framework throughout most lessons and the children seem to like it and understand it. [Teacher E]

Finally, a review of a subset of student intervention workbooks was conducted to track level of completion. All lessons involved a workbook component. Workbooks were evaluated on the 0–4 scale by two teacher-researchers to determine the extent of program completion based on written evidence of activities. Teacher-researchers had a 100% agreement rate on the completion level. Mean scores for the fidelity measures of adherence to the program, evidence of the pedagogical approach and completion of program are reported in Table 4.

Students in the control group used standard curriculum resources in covering mathematics topics from Australian national curriculum over the same three-week period. Their topics included data collation and analysis, geometric and number patterns, measurement, and number concepts.

Results

Given that the research design contained nesting structures, a multilevel (hierarchical) modeling approach was adopted to analyze group differences on pretests and treatment gains. We analyzed group differences on pretests using a two-level model (students within classrooms) with conditions dummy coded (1 = intervention and 0 = control). A two-level model was also used to analyze pretest-posttest gains; with condition groups similarly dummy coded to

Table 4. Mean scores for fidelity measures for the intervention cohort.

Fidelity Measure	Mean Score
Adherence to the program structure (Observations)	3.33
Evidence of the pedagogical approach (Observations and Self-reflection)	3.31
Completion of program (Workbook monitoring)	3.89

Scores rated on a five-point scale, with 4 as the highest score determine the direct effects of the interaction. Group means and standard deviations for spatial reasoning and mathematics are represented in Figures 5 and 6 respectively.

Screening analysis

Due to the quasi-experimental nature of the research design, screening analysis was performed to ensure equivalence of the intervention and control groups at pre-test. Results from the hierarchical pretest model revealed no difference between mean scores of control and intervention groups on SRI pretest $t(17) = .244$, $p = .81$ or Math pretest $t(17) = .244$, $p = .55$.

Treatment effects on pre-post gains

Spatial reasoning performance

Results from the hierarchical linear models for pretest-posttest gains revealed gain scores greater than 0 for each group on the Spatial Reasoning Instrument (SRI) (see Figure 5 for pretest and posttest scores) as is typical when students take a spatial test for the second time (Uttal et al., 2013). Critically, as hypothesized, the intervention groups improved more than the control groups. On average, students in the intervention group gained 1.99

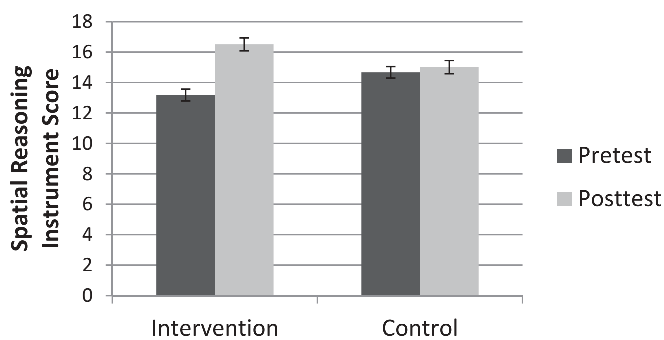


Figure 5. Pretest and posttest scores on the SRI for the intervention and control groups (error bars show standard errors).

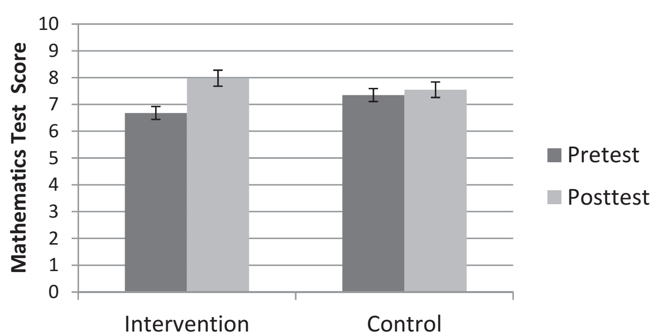


Figure 6. Pretest and posttest scores on the mathematics test for the intervention and control groups (error bars show standard errors).

more points on the SRI than did students in the control groups $t(17) = 4.59$, $p = .04$. By taking the square root of the total estimated variance as a pooled standard deviation, the treatment effect size (Cohen's d) was $d = .40$.

Hierarchical models were conducted to determine spatial skill improvement from the treatment program. On average, students in the intervention group had statistically significant increases in performance compared to the control group for both Spatial Visualization $t(16) = 7.69$, $p = .013$, $d = .41$ and Spatial Orientation $t(16) = 4.80$, $p= .041$, $d = .43$. There was no statistically significant difference in performance across the groups for the Mental Rotation construct $t(14) = 0.52$, $p = .48$. See Table 5 for descriptive statistics by spatial construct.

Mathematics performance

Results from the hierarchical linear models for pretest-posttest gains revealed gain scores greater than 0 for each group on the Mathematics test (see Figure 6 for pretest and posttest scores). On average, students in the intervention group gained 1.17 score points more than the control group on Math $t(17) = 6.95$, $p = .016$. Again, taking the square root of the total estimated variance as a pooled standard deviation, the effect size was $d = .39$. This indicates that, as hypothesized, the spatial training transferred to mathematics performance.

Additional hierarchical models were conducted to determine specific math content improvement from the treatment program. On average, students in the intervention group had statistically significant increases in performance in relation to the control group for both Geometry-based problems $t(17) = 5.92$, $p = .025$, and Mathematics word problems $t(17) = 6.11$, $p = .023$. There were no statistically significant differences in performance across the groups for Non-geometry graphics problems $t(17) = 2.95$, $p = .67$. See Table 6 for descriptive statistics by math content categories. Approximate treatment effect sizes

Table 5. Pretests, posttests and pretest-posttest gains by spatial construct.

	Intervention			Control		
	Pretest	Posttest	Gain	Pretest	Posttest	Gain
Measure	M *(SD)*	M *(SD)*	M *(SD)*	M *(SD)*	M *(SD)*	M *(SD)*
Spatial Visualization	3.83 *(1.84)*	4.69 *(2.10)*	.89 *(1.61)*	3.77 *(1.91)*	3.98 *(1.91)*	.21 *(1.68)*
Spatial Orientation	6.39 *(2.23)*	7.32 *(1.82)*	.95 *(1.58)*	6.78 *(2.06)*	7.03 *(2.09)*	.27 *(1.56)*
Mental Rotation	3.95 *(2.11)*	4.48 *(2.23)*	.55 *(2.00)*	4.35 *(2.10)*	4.76 *(2.23)*	.41 *(1.89)*

Intervention group N = 177, control group N = 150. Spatial Visualization = 10 questions, Spatial Orientation = 10 questions, Mental Rotation = 10 questions.

Table 6. Pretests, posttests and pretest-posttest gains by math categories.

	Intervention			Control		
	Pretest	Posttest	Gain	Pretest	Posttest	Gain
Category	M *(SD)*	M *(SD)*	M *(SD)*	M *(SD)*	M *(SD)*	M *(SD)*
Geometry-based	2.21 *(1.37)*	2.68 *(1.47)*	.47 *(1.33)*	2.64 *(1.52)*	2.71 *(1.46)*	.07 *(1.24)*
Word Problem	1.62 *(1.33)*	2.09 *(1.36)*	.48 *(1.17)*	1.64 *(1.42)*	1.67 *(1.47)*	.03 *(1.20)*
Non-geometry Graphic	2.84 *(1.43)*	3.03 *(1.44)*	.19 *(1.25)*	3.00 *(1.52)*	3.28 *(1.61)*	.18 *(1.31)*

Intervention group N = 177, control group N = 150. Geometry-based = 7 questions, Word Problem = 7 questions, Non-geometry Graphic = 6 questions.

(Cohen's d) for the Geometry-based problems and Word problems were d = .31 and d = .39 respectively.

Discussion

The six-hour spatial visualization program produced significant gain scores for the intervention group's mathematics performance compared to the control group, analyzed within a nested design. The moderate gain scores in mathematics performance were exhibited in tasks associated with geometry and word-based math problems. The visualization program also revealed a significant performance difference in favor of the intervention group for general spatial reasoning performance. Although the content of the intervention program was associated with only spatial visualization activities, there were significant improvements in students' gain scores for spatial orientation as well as spatial visualization. The performance increases in the intervention group's mathematics and spatial performance occurred within a relatively limited timeframe of three weeks. The effect sizes for math and spatial reasoning (.39 and .40 respectively) were equivalent to a year's expected academic growth for classroom-based education programs for students of this age (Hill, Bloom, Black, & Lipsey, 2008).

Improvement in spatial reasoning

Although the program focused on spatial visualization only, intervention groups gain scores improved across two of the three spatial constructs compared to control groups; namely, spatial visualization and spatial orientation. Elsewhere it has been suggested that spatial visualization evokes aspects of mental rotation and spatial orientation (Mix & Cheng, 2012; Mix et al., 2016) while also encompassing more complex, often multi-step mental processes and transformations (Hegarty & Waller, 2004; Kozhevnikov & Hegarty, 2001; Tversky, 2004, 2011). For students of this age, visualization skills are particularly beneficial when decoding orientation tasks, since students often report visualizing movement to consider different perspectives (Samsudin, Rafi, & Samad Hanif, 2011). It is plausible that the spatial visualization training encouraged students to visualize more effectively, which enabled more effective performance of orientation tasks. By contrast, mental rotation processes require movement of an object or shape rather than movement around objects. Most of the mental rotation tasks required the rotation of whole objects around fixed points rather than the mental transformation of an object in parts, such as paper folding, which tend to require higher levels of spatial visualization (Wright et al., 2008).

Improvement in math performance

Students who completed the intervention program showed significant performance increases on the posttest mathematics assessment compared to the control group. Notably, the effect size for this improvement (.39) was similar to that found by Lowrie et al. (2017) (.40) after a ten-week spatial training intervention. The students in the intervention group showed significant improvements on geometry problems and word problems. However, they failed to improve on problems relying on the interpretation of graphics. Given the strong association between non-verbal reasoning and graphics task performance (see Lowrie & Diezmann, 2007, 2011), this finding is somewhat surprising.

With respect to the student improvements on the geometry tasks, this finding supports previous research regarding the impact of spatial training on math performance (Lowrie et al., 2017). Geometric reasoning requires the transformation and manipulation of spatial properties (Clements & Battista, 1992), while providing the supportive scaffolds to access more complex geometric concepts (Mamolo et al., 2015). The respective intervention topics required the transformation of spatial objects across multiple representations.

One plausible explanation for the effect of training on word problem solving is that the spatial training enabled students to construct more useful spatial representations of the information in word problems. Hegarty and Kozhevnikov (1999) distinguished between two types of visual-spatial representations of a problem, schematic representations that encode the important information necessary to solve the problem and pictorial representations that encode the visual appearance of the entities described in the problem. They found that use of schematic representations was linked to spatial visualization and to success in problem solving (Hegarty & Kozhevnikov, 1999). The spatial visualization activities within the intervention program encouraged students to visualize, and it is possible that through this visualization practice (see Figure 7), they learned to construct efficient internal representations of problems. This internal encoding was likely to support the students in solving the word mathematics problems (see Battista, 2007).

In a similar vein, Lowrie (2012) distinguished between encoding and decoding processes required to solve spatial and mathematics problems, proposing that mathematics assessments have begun to shift from an emphasis on problems requiring the construction of novel spatial representations (encoding) to decoding of graphics provided in the problem, and that such a shift has exposed an underdeveloped understanding of how spatial reasoning and mathematics performance are interrelated. The students in the intervention group were encouraged to represent their thinking on paper as they progressed through the student-centered approach (see Figure 8). This type of external encoding was likely to encourage general heuristics such as "drawing diagrams" when encountering novel or challenging word problems (see Lowrie & Clements, 2001). Bresciani and Eppler (2009) drew similar conclusions, and furthermore proposed that graphic depictions or representations of information may in fact impede successful communication of information and appropriate use of spatial reasoning.

Visual displays and graphics may require decoding processes that do not immediately relate to spatial reasoning and mathematics performance (Bresciani & Eppler, 2009; Mix & Cheng, 2012). Visual displays such as graphs are highly conventional and in fact, math

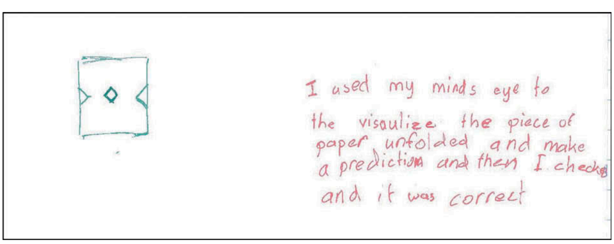


Figure 7. Internal encoding student work sample.

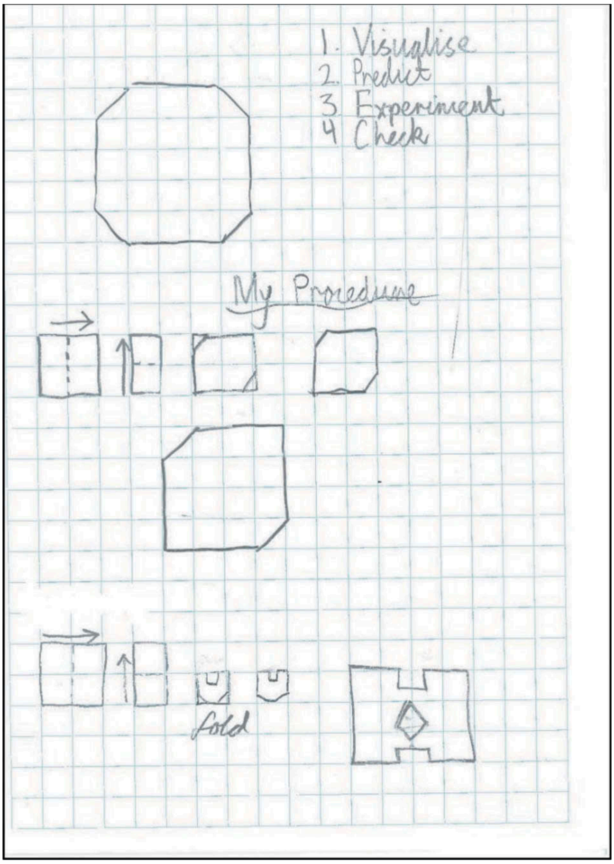


Figure 8. External encoding student work sample.

problems that contain embedded graphics require the interpretation of several specific graphic conventions such as keys, legends and labels, many of which need to be taught explicitly (Lowrie, Diezmann, & Logan, 2012). Spatial reasoning cannot compensate for lack of knowledge of the basic conventions of a graphic.

Nevertheless, spatial reasoning is generally a good predictor of student performance on mathematics tasks that contain embedded graphics, especially with students of this age (Lowrie & Diezmann, 2007). The present study has demonstrated that encoding techniques critical for the representation and interpretation of some math problems are rapidly developed through spatial visualization training. Students of this age may require a longer intervention program for such improvements to be realized for math tasks that require the decoding of graphics or training that is more focused on the conventions of graphics.

Future directions-limitations

This investigation produced comparable outcomes to that of Lowrie et al. (2017), with a more targeted, and shorter, intervention program. We have evidence that the intervention program is working from both the classrooms teachers' design of lessons and the students' engagement with these activities – in different schools, with teachers drawn from different contexts. It would be worthwhile to capture changes in the classroom teachers' discipline knowledge as they engage with the professional development aspects of the design and indeed as they implement the program in future studies. These data would allow for insights to be made about targeted professional development (PD) – with those teachers with low spatial reasoning afforded more opportunities for PD.

In this study ethical guidelines prohibited random assignment of teachers and classes. In spite of this limitation, the authenticity of this study with standard classroom demands provides insight into real world outcomes (Stieff & Uttal, 2015). In this instance the control classrooms carried on with business as usual instruction. Given the intervention classrooms received PD as part of the intervention, where random allocation is not possible, future studies may explore the impact of an active control, receiving standard mathematics PD.

We acknowledge that the study fidelity is a limitation. The distances between schools was vast, which restricted our face-to-face site visits to one visit per class. Although we gathered additional evidence of the classroom teacher's engagement with the pedagogical framework and program completion, we appreciate that additional site visits would have been best practice. To increase fidelity levels in a cost-effective manner, it might be necessary to digitally record some of the teachers' lessons, which could be analyzed to support program implementation in ways that support fidelity over time. Nevertheless, we are mindful that authentic classroom experiences are difficult to replicate, with classroom teachers needing to adjust lessons to adapt to the different contexts and student cohorts – especially across vast geographic footprints. Despite these limitations, the effectiveness of this intervention within a natural context shows promise for future work.

Future studies should undertake a more systematic analysis of students' reflections and engagement with the learning activities. Such qualitative depth will afford opportunities to monitor students' sense making and skill development. Finally, further investigations into the efficacy of intervention programs of different lengths provide more concrete evidence for interactions between spatial constructs during learning, and resulting changes in mathematics performance of students (Lowrie et al., 2017; Mix & Cheng, 2012; Mix et al., 2016). Future studies should seek to determine the durability of spatial training programs, with designs that measure student performance beyond the immediate impact of program completion.

Conclusion

This investigation provides further evidence for the effectiveness of a novel spatial reasoning intervention program, embedded within a pedagogical learning framework (ELPSA). In particular, findings indicate that a program considerably shorter than that employed by Lowrie et al. (2017) can be successful, and that a focus on one spatial construct for the entire intervention program may significantly improve not only mathematics performance, but also performance on other spatial constructs. The shorter length of the program provides opportunities for teachers to benefit their students in a shorter period of time. Although the program was only six hours in duration, effect size improvements on the math test and geometry problem and word problems sub-categories were moderately high.

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