Algorithms

for

Symmetry Analysis

by

Shih-lung Tracy Huang

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Abstract

We consider Lie point symmetry analysis of differential equations (DEs). For a family of DE systems containing arbitrary elements, the problem of symmetry classification can be solved algorithmically using a differential reduction & completion (DRC) algorithm applied to the determining equations of the symmetry vector fields. DRC algorithms such as Reid and Wittkopf's RIF split the determining equations into a number of cases. A family of DEs may additionally have some equivalence transformations which map DEs to other DEs within the same family. Case splittings of the symmetry classification should be invariant under the action of this equivalence group.

In this thesis, we give a new procedure for testing case splittings for invariance under the equivalence group action. The procedure uses the Lie infinitesimal technique and works on the level of determining equations. It is based on a method of computing prolongations of vector fields whose infinitesimals satisfy given determining equations. Our procedure does not need to know the equivalence group or the equivalence vector fields. The process is algorithmic and has been implemented as a package in the computer algebra system Maple. This package is to assist the existing DRC package `rifsimp` (which uses RIF algorithm) to improve classifying symmetries. We illustrate use of the package by applying it to symmetry classification of the 1+1 Richards equation and linear hyperbolic equations.

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