

Algorithms
for
Symmetry Analysis

by

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Abstract

We consider Lie point symmetry analysis of differential equations (DEs). For a family of DE systems containing arbitrary elements, the problem of symmetry classification can be solved algorithmically using a differential reduction & completion (DRC) algorithm applied to the determining equations of the symmetry vector fields. DRC algorithms such as Reid and Wittkopf's RIF split the determining equations into a number of cases. A family of DEs may additionally have some equivalence transformations which map DEs to other DEs within the same family. Case splittings of the symmetry classification should be invariant under the action of this equivalence group.

In this thesis, we give a new procedure for testing case splittings for invariance under the equivalence group action. The procedure uses the Lie infinitesimal technique and works on the level of determining equations. It is based on a method of computing prolongations of vector fields whose infinitesimals satisfy given determining equations. Our procedure does not need to know the equivalence group or the equivalence vector fields. The process is algorithmic and has been implemented as a package in the computer algebra system Maple. This package is to assist the existing DRC package `rifsimp` (which uses RIF algorithm) to improve classifying symmetries. We illustrate use of the package by applying it to symmetry classification of the 1+1 Richards equation and linear hyperbolic equations.

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Contents

1	Introduction	5
2	Symmetry Analysis of DEs	11
2.1	Symmetry Analysis	13
2.1.1	Lie Point Symmetry Analysis	18
2.2	Symmetry Classification	22
2.3	Equivalence Transformations	25
3	Computer Algebra in Symmetry Analysis	35
3.1	Symmetry Analysis Packages	36
3.2	Differential Reduction & Completion Algorithms	37
3.2.1	The RIF Algorithm	41
3.3	Symmetry Classification Using RIF	51
3.3.1	Symmetry Classification Using Rifsimp	54
3.4	Results from Commutative Algebra	57
4	Invariance Checking from Determining Equations	61
4.1	Projection of Equivalence Group Action	62
4.2	Issues with Symmetry Condition	67
4.3	Invariance Using Determining Equations	72
4.4	Invariance Checking in Symmetry Classification	81

Contents

4.4.1	Label pivots from classification tree	82
4.4.2	Guide RIF during classification	88
5	Implementation of Invariance Checking Method	91
5.1	Required Implementations	92
5.2	Symmetry Classification Package	95
5.2.1	Storage Structure for DE System	97
5.2.2	Pre-step methods	99
5.2.3	Rifsimp with the ICM method	100
5.2.4	Display Procedure	103
5.2.5	Front-end procedure	103
5.3	Examples	104
5.3.1	1+1 Richards Equation	104
5.3.2	Linear Hyperbolic Equation with Laplace Invariants	110
6	Conclusion	115
	References	121
A	The SymmetryClassification Package	129
	SymmetryClassification Overview	130
	pdeRecord	134
	newPDESys	141
	detEqsForSymm	144
	detEqsForEquiv	147
	newProlongation	150
	AddInvtInfo	156
	SymmetricRifsimp	159
	CasePlot	162

Contents

<code>classifySymmetry</code>	168
B Published work arising from the thesis	173