

**AN INVESTIGATION OF PRIMARY TEACHERS'
MATHEMATICAL PEDAGOGICAL CONTENT
KNOWLEDGE**

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Abstract

In an era of educational reform, investigating teachers' pedagogical content knowledge has implications for many involved in education, from policy makers and curriculum designers to those in teacher education. This thesis proposed a model, designed by the researcher, used to examine Shulman's (1986) theory of pedagogical content knowledge. In particular, it addressed primary teachers' pedagogical content knowledge required for teaching measurement.

By examining teachers' mathematics pedagogical content knowledge a greater understanding of teachers' professional knowledge was gained enabling improvement of teacher quality, by being able to identify more clearly individual teacher's needs for professional development. This study addressed four specific research questions.

How evident is the teacher's depth of mathematical knowledge of measurement within their teaching? How do teachers show that they understand and address the needs of students when teaching? How do teachers demonstrate their general pedagogical knowledge when teaching? How is a teacher's knowledge and practice impacted by other factors when teaching and what are these major factors?

A qualitative research model was used in which four teachers of Years Three and Four participated, providing four individual case studies. Each teacher was interviewed at the commencement of the study, was observed and recorded throughout their teaching of a sequence of measurement lessons, interviewed prior to and following each lesson, and finally responded to a reflective questionnaire two weeks after the sequence of lessons had concluded.

Due to the extensive nature of the data, a series of vignettes was written, based upon

identified teaching episodes, significant to addressing the research questions. These vignettes contributed to the cross case analysis (Yin, 2010), along with the other data. The study found that the teachers' knowledge varied considerably in each of the areas of knowledge of teaching, knowledge of students and knowledge of mathematics. Consequently, the teachers were rated differently in relation to their pedagogical content knowledge, ranging from very weak to strong. These differences were examined in terms of the model, providing evidence that the model effectively explained variations in teachers' pedagogical content knowledge. Factors such as self-efficacy, teacher beliefs and the culture of the school were also shown to influence each teacher's pedagogical content knowledge. The model was shown to be dynamic and it clearly identified how and why pedagogical content knowledge varied from one teacher to another.

This study has shown that the model used to represent pedagogical content knowledge demonstrated theoretical, methodological and diagnostic value. This study concludes with a discussion of implications for policy and practice at system level and for teacher education courses for preservice teachers. The findings of this study provide further understanding of teacher pedagogical content knowledge, which is an essential step towards improving teacher quality and teaching practice. The evidence suggests that this model could be used for further research into pedagogical content knowledge beyond the teaching of measurement.

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